Antenna Theory

$$\vec{E}^T = \vec{E}^i + \vec{E}^s \qquad \hat{z}_0 \hat{y}_0$$

$$\vec{H}^T = \vec{H}^i + \vec{H}^s$$

Incident problem

$$-\vec{\nabla} \times \vec{E}^i = \hat{z}_0 \vec{H}^i + \vec{M}^i$$

$$\nabla \times H^i = \hat{y}_0 E^i + \vec{J}^i$$

Scattered Problem

$$-\vec{\nabla} \times \vec{E}^s = \hat{z}_0 \vec{E}^s + \vec{M}^s$$

$$\vec{\nabla} \times \vec{H}^s = y_0 \vec{E}^s + \vec{J}^s$$

$$\vec{M}^s = (\hat{z}_T - \hat{z}_0) H^T = j\omega(\mu_T - \mu_0) H^T$$

$$\vec{J}^s = (\hat{y}_T - \hat{y}_0) E^T = j\omega(\varepsilon_T - \varepsilon_0) E^T + \sigma_T E^T$$

Assume the antenna is a non-magnetic <u>metal</u> object

$$\mu_T = \mu_0 \quad \therefore \vec{M}^s = 0$$

$$\therefore \vec{F}^s = 0$$

$$\varepsilon_T = \varepsilon_0$$

$$\vec{J}^s = \sigma_T \vec{E}^T$$

$$\nabla^2 \vec{A} + \omega^2 \mu_0 \varepsilon \vec{A} = -J$$

$$\omega^2 \mu_0 \varepsilon_0 = k^2$$

Green's Function

$$\nabla^2 G + \omega^2 \mu_0 \varepsilon_0 G = I\vec{l}\, \delta(\vec{r}-\vec{r}\,')$$

$$\delta(r-r') = \delta(x-x_i')\,\delta(y-y')\,\delta(z-z')$$

$$G = \frac{I\vec{l}}{4\pi} \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}$$ (outgoing solution)

$$\vec{A}(\vec{r}) = \frac{1}{4\pi} \iiint_V \frac{\vec{J}(\vec{r}\,')\, e^{-jk|r-r'|}}{|\vec{r}-\vec{r}\,'|}\, dv'$$

$$\vec{A}_s(\vec{r}) = \frac{1}{4\pi} \iiint_V \sigma_T(r')\, \vec{E}^T(r') \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\, dv'$$

$\vec{A}_s(\vec{r})$ $\sigma \Rightarrow \infty$ $\sigma_T E^T$ becomes a true surface current

$$A_s(\vec{r}) = \frac{1}{4\pi} \oiint_{\text{surface}} J_{\text{surface}}(E^T(r')) \frac{e^{-jk|r-r'|}}{|r-r'|}$$

$$\vec{J}_{\text{surface}} = \hat{n} \times \vec{H}_{\text{surface}}$$

We still don't know what $\vec{H}_{surface}$ is so we'll <u>guess</u>!

1) Since it is an integral Equation, it is tolerant of error's in our guess's

2) We can put reasonable bounds on our guess's such as the current at the end of a wire is zero.

3) Do not lose sight of the fact that an antenna just redirects an incident field into a scattered field. Since the electric field on the surface of the antenna is zero, the surface currents on the antenna are not the source of the radiating power!

However the surface currents form an intermediate step that is convenient for calculations.

The Ideal Dipole of infinitely short length

$$\vec{A} = \frac{I \Delta z \, \vec{z} \, e^{-jk|\vec{r}|}}{4\pi |\vec{r}|} \qquad r = |\vec{r}|$$

Given that $\vec{r}' \simeq 0$ since the dipole is so small

$$\vec{H} = \vec{\nabla} \times \vec{A} = \nabla \times (A_z \hat{z})$$

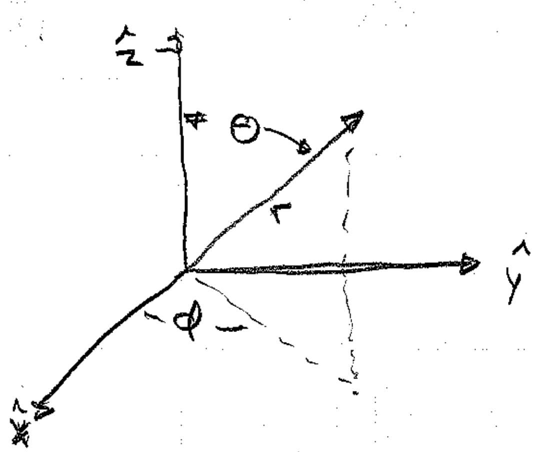
$$\vec{\nabla} \times (f \vec{G}) = \vec{\nabla}(f) \times \vec{G} + f(\vec{\nabla} \times \vec{G})$$

$$\vec{H} = (\vec{\nabla} A_z \times \hat{z}) + A_z \underbrace{\cancel{(\vec{\nabla} \times \vec{z})}}_{0}$$

$$\vec{H} = (\nabla A_z) \times \vec{z}$$

$$\vec{H} = \frac{I \Delta z}{4\pi} \nabla \left(\frac{e^{-jkr}}{r} \right) \times \hat{z}$$

Spherical Coordinates



$$x = r \sin\theta \cos\phi$$

$$\hat{y} = r \sin\theta \sin\phi$$

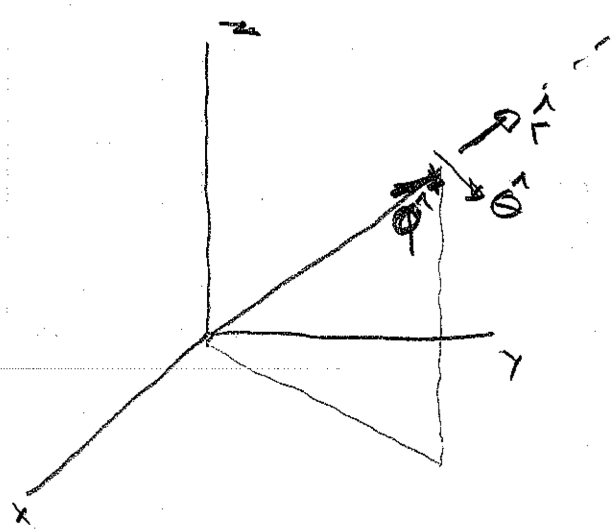
$$\hat{z} = r \cos\theta$$

$$\vec{\nabla} \vec{g} = \frac{\partial g}{\partial x}\hat{x} + \frac{\partial g}{\partial y}\hat{y} + \frac{\partial g}{\partial z}\hat{z}$$

$$= \hat{r}\frac{\partial g}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial g}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial g}{\partial \phi}$$

$$\vec{H} = \frac{I\Delta z}{4\pi}\frac{\partial}{\partial r}\left(\frac{e^{-jkr}}{r}\right)\hat{r}\times\hat{z}$$

$$= \frac{I\Delta z}{4\pi}\left[\frac{-jke^{-jkr}}{r} - \frac{e^{-jkr}}{r^2}\right]\hat{r}\times\hat{z}$$



$$\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$$

(check at $\theta = 0$

$\theta = \frac{\pi}{2}$

$$\hat{r} \times \vec{r} = 0$$

$$\hat{r} \times \hat{\theta} = \hat{\phi}$$

$$\hat{r} \times \hat{z} = -\hat{\phi} \sin\theta$$

$$\vec{H} = \frac{I \Delta z}{4\pi} \left[\frac{jk}{r} + \frac{1}{r^2} \right] e^{-jkr} \sin\theta \, \hat{\phi}$$ (makes sense)

$$\vec{E} = \frac{1}{j\omega\varepsilon} \vec{\nabla} \times \vec{H}$$

far field, near field

$$\vec{E} = \frac{I \Delta z}{4\pi} \left[j\frac{\omega\mu}{r} + \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{r^2} + \frac{1}{j\omega\varepsilon r^3} \right] e^{-jkr} \sin\theta \, \hat{\phi}$$

$$+ \frac{I \Delta z}{2\pi} \left[\sqrt{\frac{\mu}{\varepsilon}} \frac{1}{r^2} + \frac{1}{j\omega\varepsilon} \frac{1}{r^3} \right] e^{-jkr} \cos\theta \, \hat{r}$$

near field

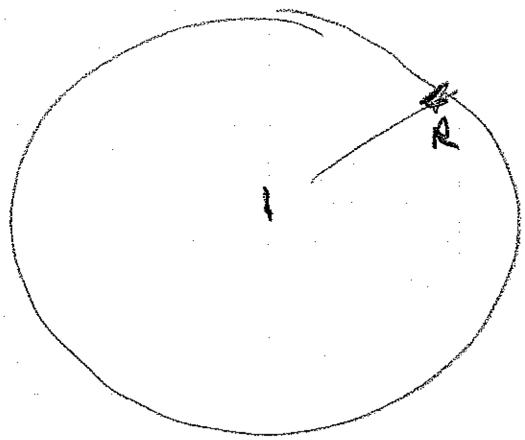
Far Field is the only thing left as $r \to \infty$

$$\vec{E} = \frac{I \Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r} \sin\theta \, \hat{\theta}$$

$$\vec{H} = \frac{I \Delta z}{4\pi} jk \frac{e^{-jkr}}{r} \sin\theta \, \hat{\phi}$$

$$\frac{E_\theta}{H_\phi} = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

$\angle$ Characteristic of the far field.



$R \to \infty$

$$P_f = \frac{1}{2} \iint_S \vec{E} \times \vec{H}^* \cdot d\vec{s}$$

$$\hat{\theta} \times \hat{\phi} = \hat{r}$$

$$d\vec{s} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

$$P_f = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \int_0^{2\pi}\int_0^{\pi} j\omega\mu \frac{e^{-jkr}}{r} \sin\theta \, (-jk) \frac{e^{+jkr}}{r} \sin\theta \, ds$$

$$\frac{d\vec{s}}{r^2} = \sin d\theta \;\; d\theta \, d\phi$$

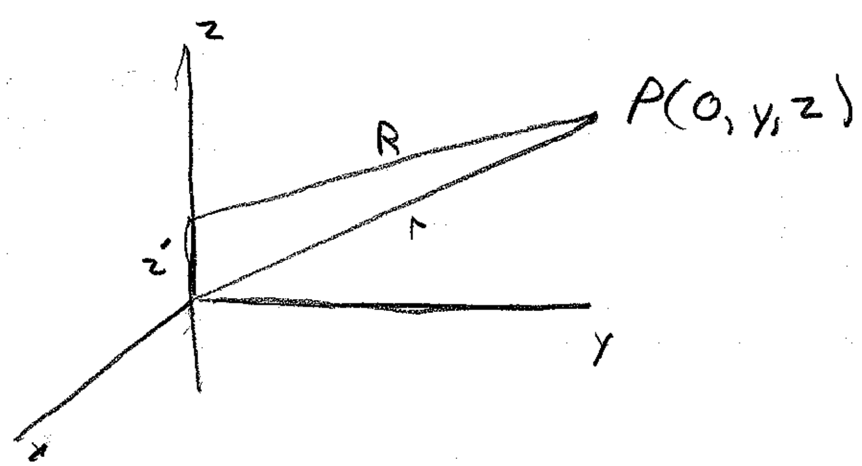
P_f is not a function of R !

$$P_f = \frac{\omega\mu k}{12\pi}(I\Delta z)^2$$

Far field Gives the power flowing <u>away</u> from the antenna.

The Near field represents the energy stored near the antenna.

<u>Extended Sources</u>



By symmetry, there should be no ϕ dependence so we can look at any ϕ angle

$$r^2 = y^2 + z^2$$
$$z = r\cos\theta$$
$$y = r\sin\theta \qquad (\text{for } \varphi = \tfrac{\pi}{2})$$

$$R^2 = y^2 + (z-z')^2$$

$$= \underbrace{y^2+z^2}_{r^2} - \underbrace{2zz'}_{2r\cos\theta\, z'} + (z')^2$$

$$R^2 = r^2 + \left[-2r\cos\theta\, z' + (z')^2\right]$$

$$R^2 \approx r^2 - 2r\cos\theta\, z'$$

$$\cong r^2\left(1 - \frac{2\cos\theta}{r} z'\right)$$

$$R = r\left(1 - \frac{2\cos\theta}{r} z'\right)^{1/2}$$

$$R \cong r\left(1 - \frac{\cos\theta}{r} z'\right)$$

$$R = r - \cos\theta\, z'$$

phase term

$$A_z = \int I(z') \frac{e^{-jk(r-z'\cos\theta)}}{4\pi(r-z'\cos\theta)} dz'$$

amplitude term

$$A_z \cong \int I(z') \frac{e^{-jk(r-z'\cos\theta)}}{4\pi r} dz'$$

$$A_z = \frac{e^{-jkr}}{4\pi r} \int I(z') e^{+jkz'\cos\theta} \, dz'$$

$$H = \vec{\nabla} \times \vec{A}$$

$$H = \nabla \times \left(-A_z \sin\theta \, \hat{\theta} + A_z \cos\theta \, \hat{r}\right)$$

$$\nabla \times G = \hat{r} \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial\theta}(G_\phi \sin\theta) - \frac{\partial G_\theta}{\partial\phi}\right] = 0$$

$$+ \hat{\theta} \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial G_r}{\partial\phi} - \frac{\partial}{\partial r}(r G_\phi)\right] = 0$$

$$+ \hat{\phi} \frac{1}{r} \left[\frac{\partial}{\partial r}(r G_\theta) - \frac{\partial G_r}{\partial\theta}\right]$$

$$\vec{H} = \hat{\phi} \frac{1}{r} \left[\frac{\partial}{\partial r}\left(-r A_z \sin\theta\right) - \frac{\partial}{\partial\theta}\left(A_z \cos\theta\right)\right]$$

$$\vec{H} = \hat{\phi} \frac{e^{-jkr}}{4\pi r} \Big\{ jk\sin\theta \int I(z') e^{jkz'\cos\theta} \, dz'$$

$$- \frac{1}{r} \frac{\partial}{\partial\theta}\left[\cos\theta \int I(z') e^{jkz'\cos\theta} \, dz'\right] \Big\}$$

near field

$$\vec{H}_{far} = \hat{\phi} \, jk\sin\theta \frac{e^{-jkr}}{4\pi r} \int I(z') e^{jkz'\cos\theta} \, dz'$$

But

$$A_z = \frac{e^{-jkr}}{4\pi r} \int I(z') e^{jkz'\cos\theta} dz'$$

$$\vec{H}_{far} = \hat{\phi}(jk\sin\theta)\, A_z$$

$$\vec{E}_{far} = \frac{1}{j\omega\varepsilon} \vec{\nabla} \times \vec{H}_{far}$$

$$j\omega\varepsilon E_{far} = \hat{r}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(H_\phi \sin\theta\right)\right] \Rightarrow 0 \quad \text{near field}$$

$$- \hat{\theta}\, \frac{1}{r}\frac{\partial}{\partial r}\left(r H_\phi\right) \Big]$$

$$j\omega\varepsilon E_{far} = -\hat{\theta}\; k^2 \sin\theta \frac{e^{-jkr}}{4\pi r} \int I(z') e^{jkz'\cos\theta} dz'$$

$$E_{far} = \hat{\theta}\left(j\frac{k^2}{\omega\varepsilon}\sin\theta\right) A_z$$

$$= \hat{\theta}\, \frac{k}{\omega\varepsilon}\left(jk\sin\theta\right) A_z$$

$$\boxed{\begin{aligned} \vec{E}_{far} &= \hat{\theta}\sqrt{\frac{\mu}{\varepsilon}}\,(jk\sin\theta)\, A_z \\ \vec{H}_{far} &= \hat{\phi}\,(jk\sin\theta)\, A_z \\ A_z &= \frac{e^{-jkr}}{4\pi r}\int I(z') e^{jkz'\cos\theta} dz' \end{aligned}}$$

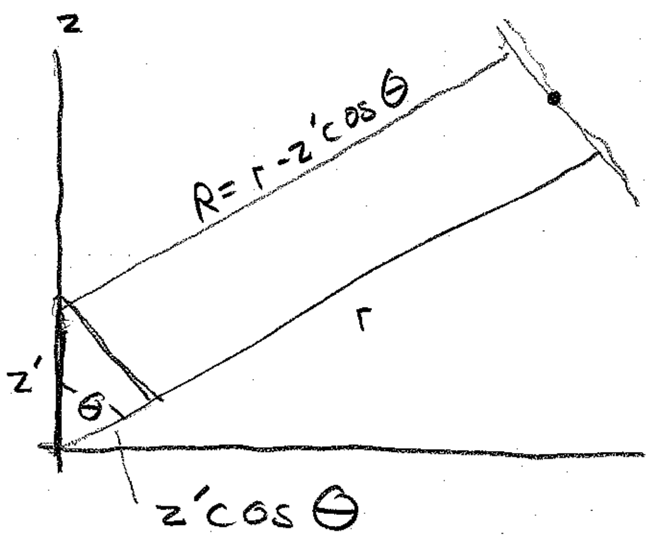
Ratio of

$$\frac{E_{Far}}{H_{Far}} = \eta$$

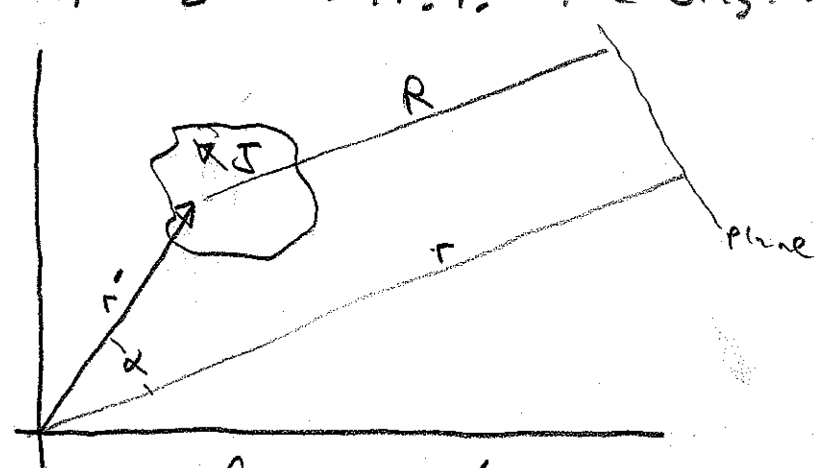
Characteristic of plane waves

Geometric interpretation of

$$e^{-jk(r - z'\cos\theta)}$$



$kz'\cos\theta$ is just the phase difference of a small current element located at z' w.r.t. the origin



$$R = r - r'\cos\alpha$$

$$r'\cos\alpha = \frac{\vec{r}' \cdot \vec{r}}{r}$$

<u>In the far field Approx</u>

$$R = r - r' \frac{\vec{r} \cdot \vec{r}'}{r r'}$$

or

$$R = r - \hat{r} \cdot \vec{r}'$$

When does the far field break down

$$R = r - z'\cos\theta + \frac{(z')^2 \sin^2\theta}{2r} \ldots\ldots$$

$$\frac{(z'_{max})^2}{2 r_{ff}} < \frac{\lambda}{16}$$ ⟸ somewhat arbitrary so there is small contribution

$$z'_{max} = \frac{L}{2}$$

$$r_{ff} > 2\left(\frac{L}{\lambda}\right) L$$

<u>Also</u>

$$r_{ff} >> L$$
$$r_{ff} >> \lambda$$

to get rid of near field