Introduction to RF for Particle Accelerators
Part 1: Transmission Lines

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Motivation

- These lectures are intended as an introduction to RF terminology and techniques
- Lectures are based on the notes for the Microwave Measurement Class that is taught at the US Particle Accelerator School

Part 1 - Transmission Lines

- Phasors
- Traveling Waves
- Characteristic Impedance
- Reflection Coefficient
- Standing Waves
- Impedance and Reflection
- Incident and Reflected Power
- Smith Charts
- Load Matching
- Single Stub Tuners
- dB and dBm
- Z and S parameters
- Lorentz Reciprocity
- Network Analysis
- Phase and Group Delay

Part 2 - RF Cavities

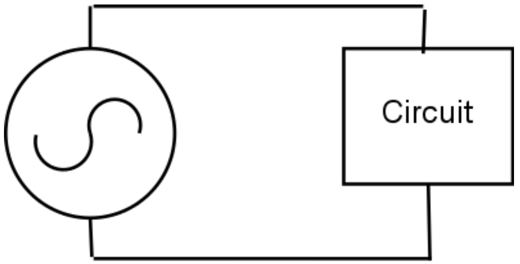
- Modes
  - Symmetry
  - Boundaries
  - Degeneracy
- RLC model
- Coupling
  - Inductive
  - Capacitive
  - Measuring
- Q
  - Unloaded Q
  - Loaded Q
  - Q Measurements
- Impedance Measurements
  - Bead Pulls
  - Stretched wire
- Beam Loading
  - De-tuning
  - Fundamental
  - Transient
- Power Amplifiers
  - Class of operation
  - Tetrodes
  - Klystrons

Part 3 - Beam Signals

- Power Spectral Density
- Spectra of bunch loading patterns
- Betatron motion
- AM modulation
- Longitudinal motion
- FM and PM modulation
- Multipole distributions

Terminology and Conventions

Sinusoidal Source



$$V(t) = V_o \cos(\omega t + \phi)$$

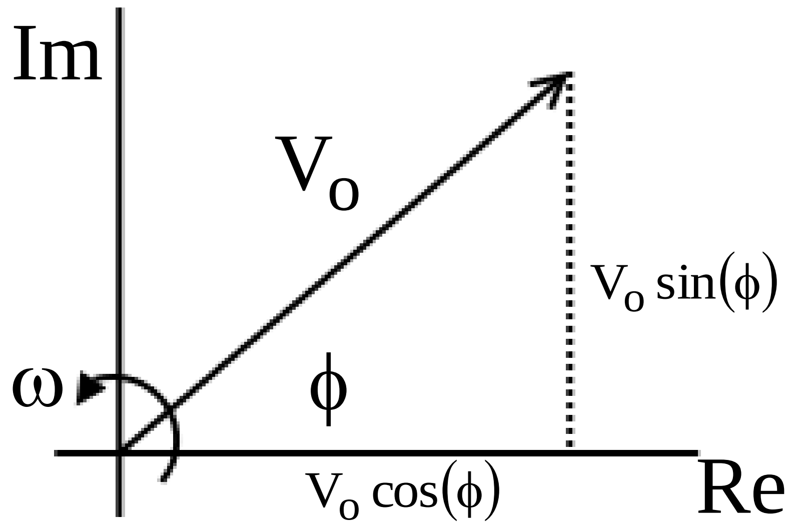
$$V(t) = \mathrm{Re}\left\{V_o e^{j(\omega t + \phi)}\right\} = \mathrm{Re}\left\{V_o e^{j\phi} e^{j\omega t}\right\}$$

$$j = \sqrt{-1}$$

$V_o e^{j\phi}$ is a complex phasor

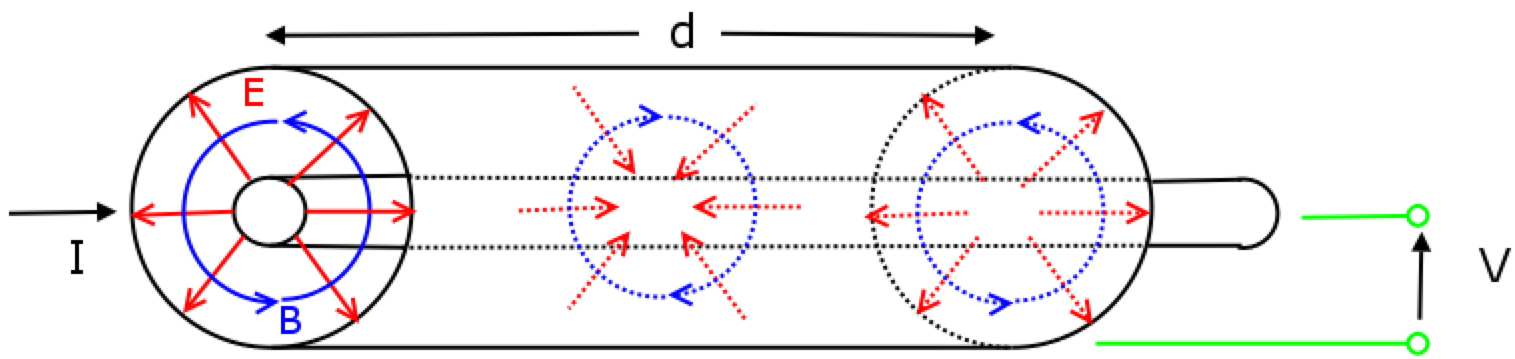
Phasors

$$V_o e^{j\phi} = V_o \cos(\phi) + jV_o \sin(\phi)$$



- In these notes, all sources are sine waves
- Circuits are described by complex phasors
- The time varying answer is found by multiplying phasors by $e^{j\omega t}$ and taking the real part

TEM Transmission Line Theory



Charge on the inner conductor:

$$\Delta q = C_l \Delta x V$$

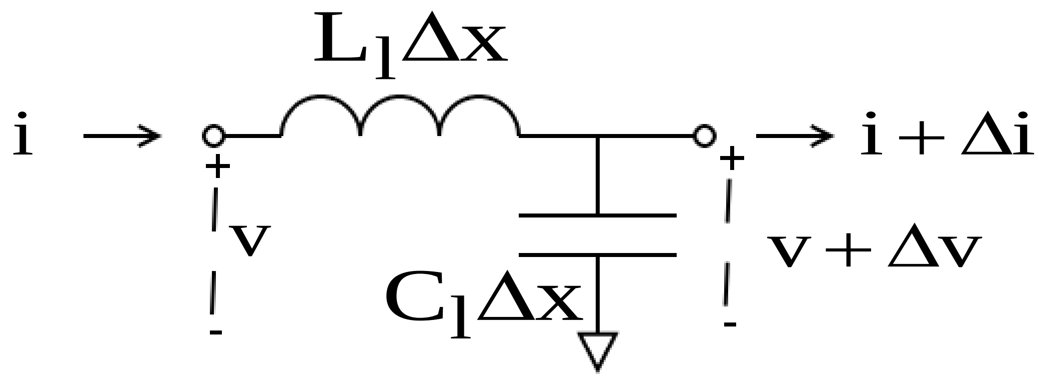
where C_l is the capacitance per unit length

Azimuthal magnetic flux:

$$\Delta \Phi = L_l \Delta x I$$

where L_l is the inductance per unit length

Electrical Model of a Transmission Line



Voltage drop along the inductor:

$$v - (v + \Delta v) = L_l \Delta x \frac{di}{dt}$$

Current flowing through the capacitor:

$$i + \Delta i = i - C_l \Delta x \frac{dv}{dt}$$

Transmission Line Waves

Limit as Δx->0

$$\frac{\partial v}{\partial x} = -L_l \frac{\partial i}{\partial t} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\frac{\partial i}{\partial x} = -C_l \frac{\partial v}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Solutions are traveling waves

$$v(t,x) = v^{+}\left(t - \frac{x}{vel}\right) + v^{-}\left(t + \frac{x}{vel}\right)$$

$$i(t,x) = \frac{v^{+}}{Z_o}\left(t - \frac{x}{vel}\right) - \frac{v^{-}}{Z_o}\left(t + \frac{x}{vel}\right)$$

v^{+} indicates a wave traveling in the +x direction
v^{-} indicates a wave traveling in the -x direction

Phase Velocity and Characteristic Impedance

vel is the phase velocity of the wave

$$\mathrm{vel} = \frac{1}{\sqrt{L_l C_l}}$$

For a transverse electromagnetic wave (TEM), the phase velocity is only a property of the material the wave travels through

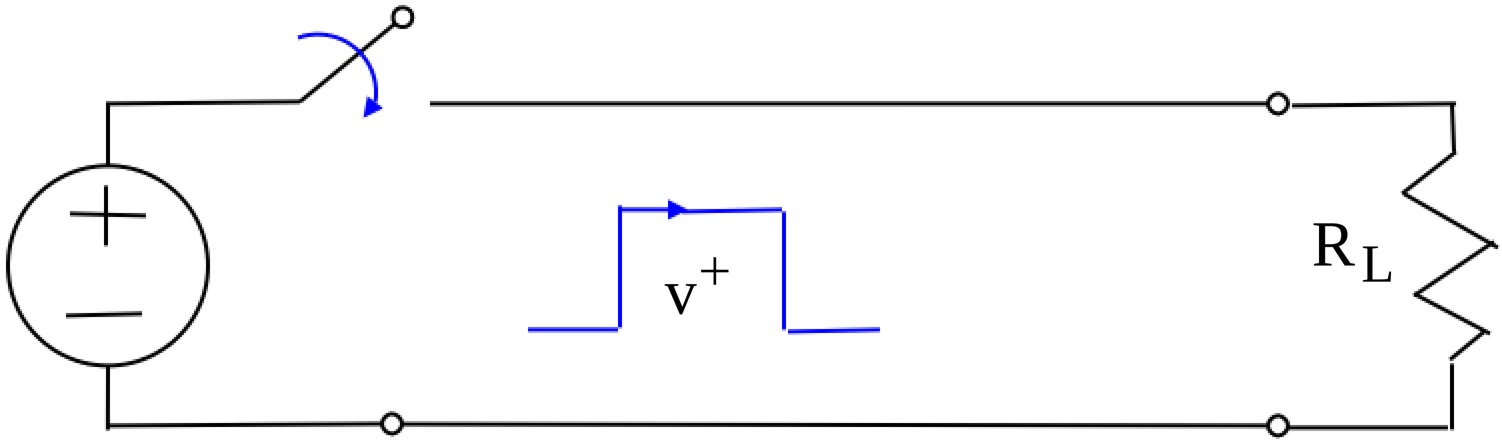
$$\frac{1}{\sqrt{L_l C_l}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

The characteristic impedance Z_o

$$Z_o = \sqrt{\frac{L_l}{C_l}}$$

has units of Ohms and is a function of the material AND the geometry

Pulses on a Transmission Line



Pulse travels down the transmission line as a forward going wave only (v^+). However, when the pulse reaches the load resistor:

$$\frac{v}{i} = R_L = \frac{v^+ + v^-}{\frac{v^+}{Z_o} - \frac{v^-}{Z_o}}$$

so a reverse wave v^- and i^- must be created to satisfy the boundary condition imposed by the load resistor

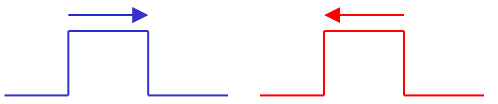
Reflection Coefficient

The reverse wave can be thought of as the incident wave reflected from the load

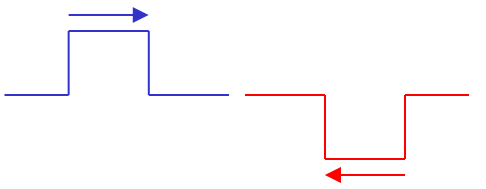
$$\frac{v^{-}}{v^{+}} = \frac{R_L - Z_o}{R_L + Z_o} = \Gamma$$ Reflection coefficient

Three special cases:

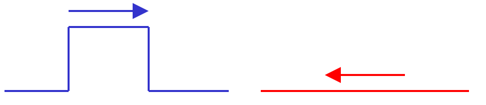
$R_L = \infty$ (open) $\Gamma = +1$



$R_L = 0$ (short) $\Gamma = -1$



$R_L = Z_o$ $\Gamma = 0$



A transmission line terminated with a resistor equal in value to the characteristic impedance of the transmission line looks the same to the source as an infinitely long transmission line

Sinusoidal Waves

Experiment: Send a **SINGLE** frequency (ω) sine wave into a transmission line and measure how the line responds

$$v^{+} = V^{+}\cos(\omega t - \beta x) = \mathrm{Re}\left\{V^{+} e^{-j\beta x} e^{j\omega t}\right\}$$

$$\frac{\omega}{\beta} = \mathrm{vel}$$ phase velocity

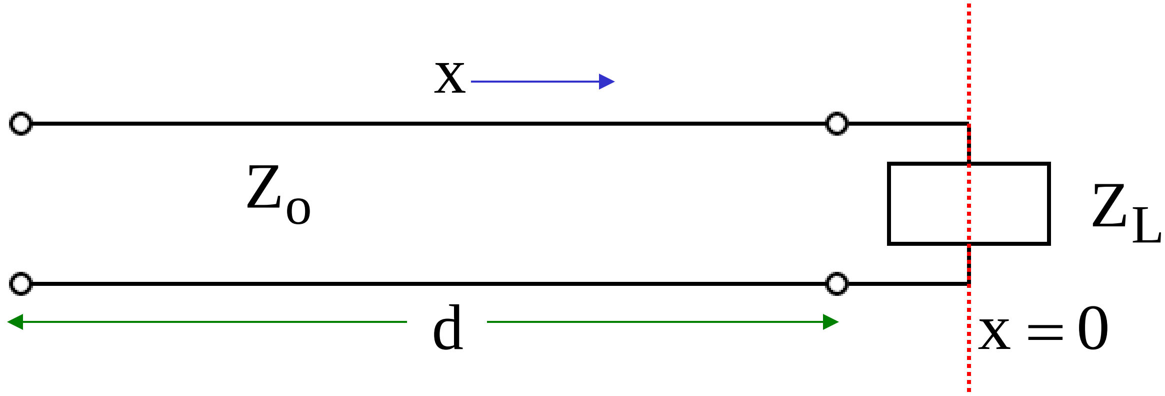
$$\beta = \frac{2\pi f}{\mathrm{vel}} = \frac{2\pi}{\lambda}$$ wave number

By using a single frequency sine wave we can now define complex impedances such as:

$$v = L\frac{di}{dt} \qquad V = j\omega L I \qquad Z_{ind} = j\omega L$$

$$i = C\frac{dv}{dt} \qquad I = j\omega C V \qquad Z_{cap} = \frac{1}{j\omega C}$$

Standing Waves



At x=0

$$V^- = \Gamma V^+ = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Along the transmission line:

$$V = V^+ e^{-j\beta x} + \Gamma V^+ e^{+j\beta x}$$

$$V = V^+ (1 - \Gamma) e^{-j\beta x} + 2V^+ \Gamma \cos(\beta x)$$

traveling wave

standing wave

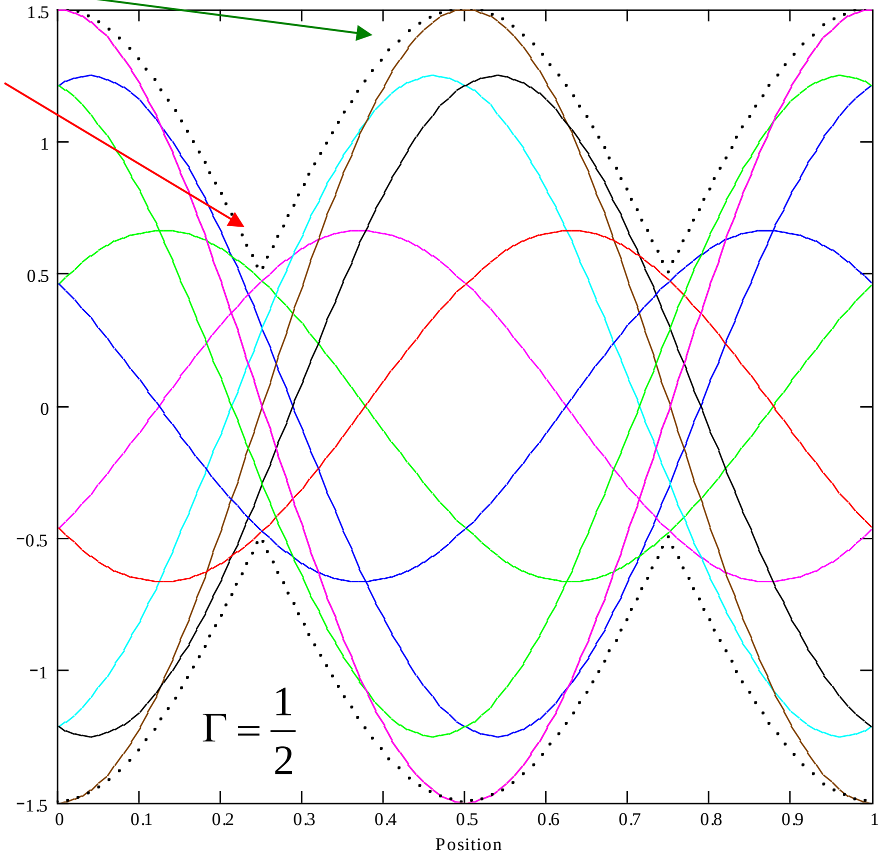
Voltage Standing Wave Ratio (VSWR)

Large voltage

Large current

$$\frac{V_{max}}{V_{min}} = \frac{V^+(1+|\Gamma|)}{V^+(1-|\Gamma|)} = \frac{(1+|\Gamma|)}{(1-|\Gamma|)}$$

$$= \mathrm{VSWR}$$

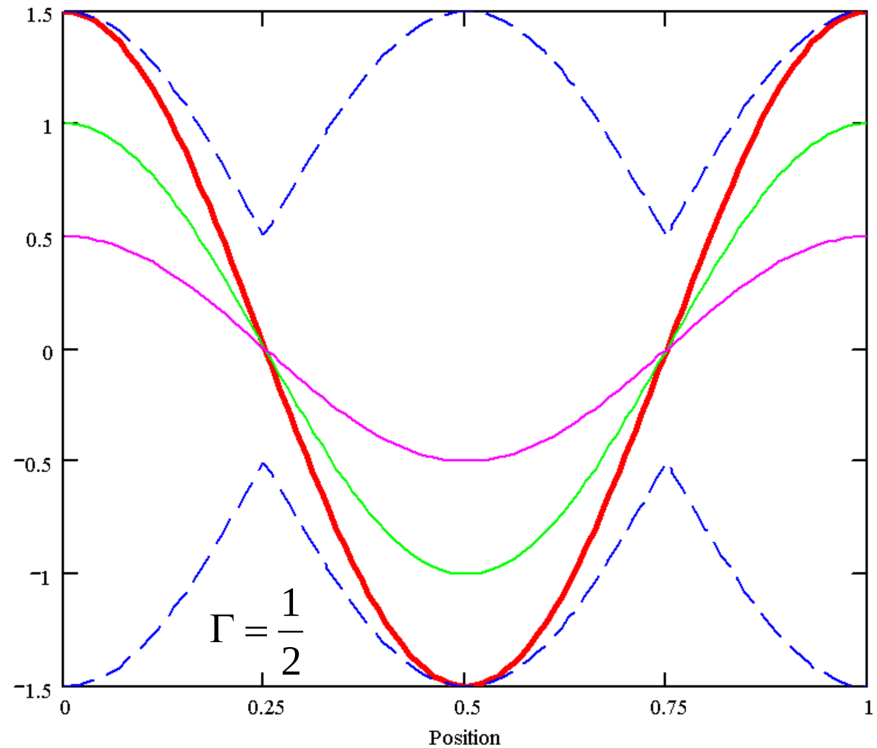


The VSWR is always greater than 1

Voltage Standing Wave Ratio (VSWR)

$$\frac{V_{max}}{V_{min}} = \frac{V^{+}\left(1+|\Gamma|\right)}{V^{+}\left(1-|\Gamma|\right)} = \frac{\left(1+|\Gamma|\right)}{\left(1-|\Gamma|\right)}$$

$$= \mathrm{VSWR}$$



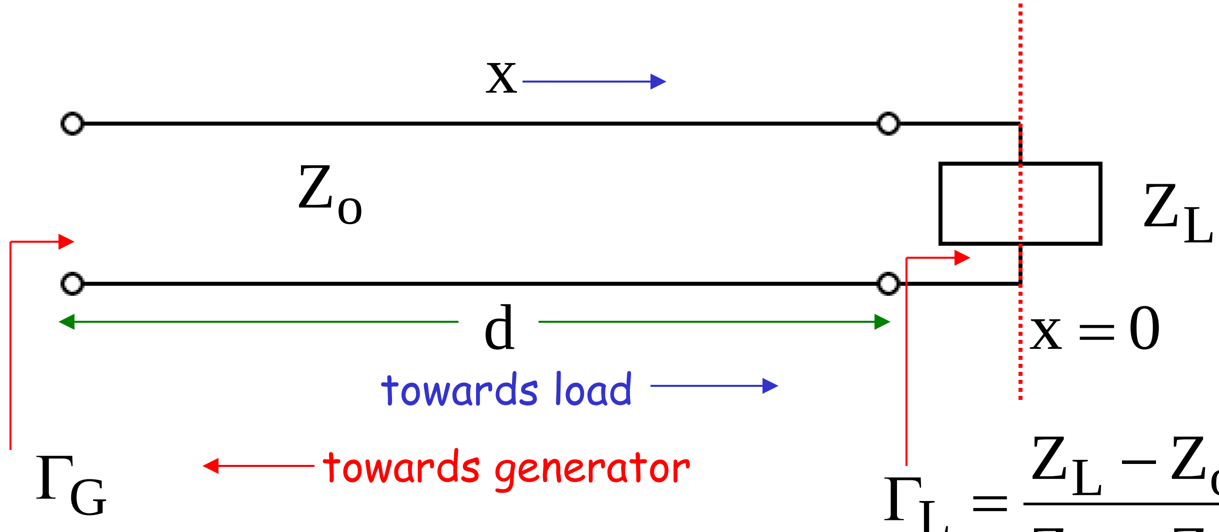
Incident wave

Reflected wave

Standing wave

The VSWR is always greater than 1

Reflection Coefficient Along a Transmission Line



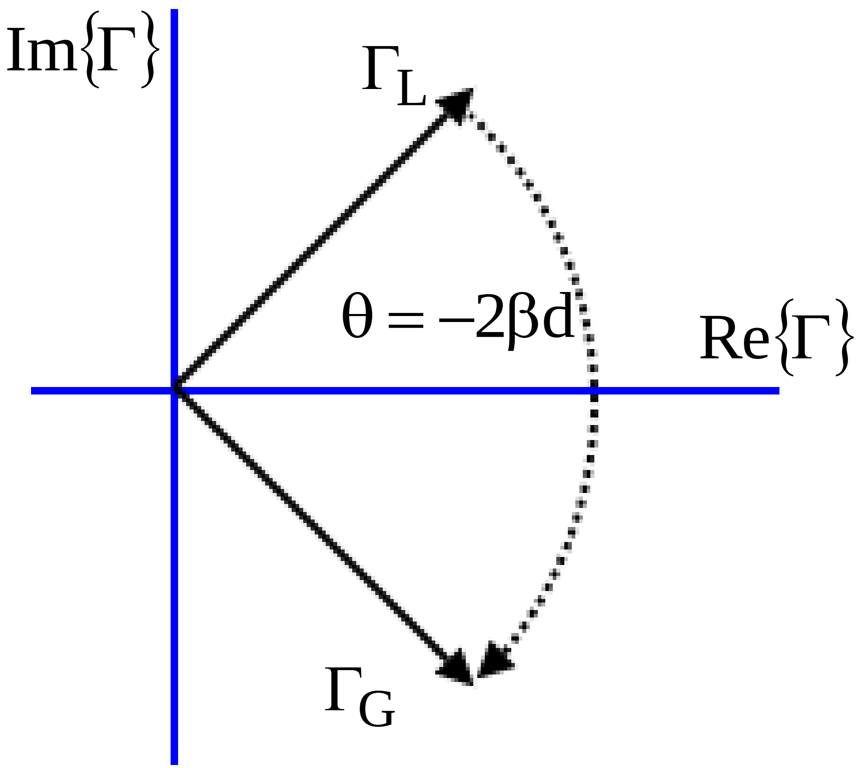
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V = V^{+} e^{-j\beta x} + \Gamma_L V^{+} e^{+j\beta x}$$

Wave has to travel down and back

$$\Gamma_G = \left.\frac{V_{\mathrm{forward}}}{V_{\mathrm{reverse}}}\right|_{\mathrm{gen}} = \Gamma_L \frac{V^{+} e^{+j\beta(-d)}}{V^{+} e^{-j\beta(-d)}} = \Gamma_L e^{-j2\beta d}$$

Impedance and Reflection



There is a one-to-one correspondence between Γ_G and Z_L

$$\Gamma_G = \frac{Z_G - Z_o}{Z_G + Z_o}$$

$$Z_G = Z_o \frac{1 + \Gamma_G}{1 - \Gamma_G}$$

$$Z_G = Z_o \frac{1 + \Gamma_L e^{-j2\beta d}}{1 - \Gamma_L e^{-j2\beta d}}$$

Impedance and Reflection: Open Circuits

For an open circuit $Z_L = \infty$ so $\Gamma_L = +1$

Impedance at the generator:

$$Z_G = \frac{-jZ_o}{\tan(\beta d)}$$

For $\beta d \ll 1$

$$Z_G \approx \frac{-jZ_o}{\beta d} = \frac{1}{j\omega C_l d}$$

looks capacitive

For $\beta d = \pi/2$ or $d = \lambda/4$

$$Z_G = 0$$

An <u>open</u> circuit at the load looks like a <u>short</u> circuit at the generator if the generator is a quarter wavelength away from the load

Impedance and Reflection: Short Circuits

For a short circuit $Z_L = 0$ so $\Gamma_L = -1$

Impedance at the generator:

$$Z_G = jZ_o \tan(\beta d)$$

For $\beta d \ll 1$

$$Z_G \approx jZ_o \beta d = j\omega L_l d$$

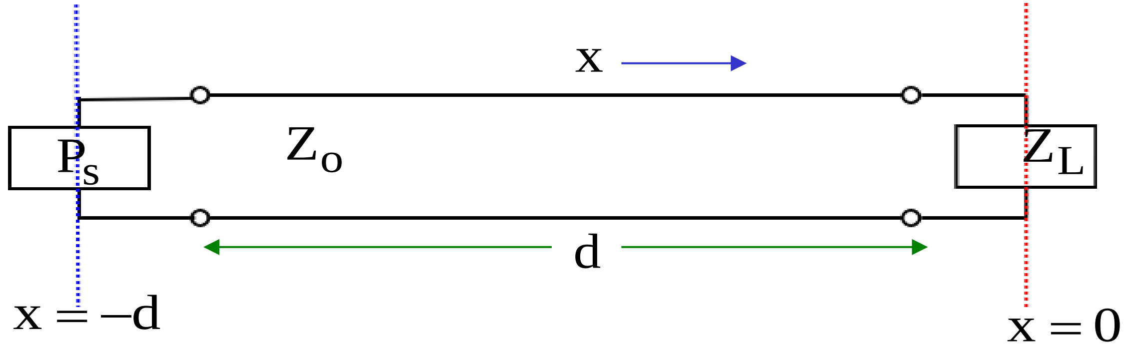
looks inductive

For $\beta d = \pi/2$ or $d = \lambda/4$

$$Z_G \to \infty$$

A <u>short</u> circuit at the load looks like an <u>open</u> circuit at the generator if the generator is a quarter wavelength away from the load

Incident and Reflected Power



Voltage and Current at the generator (x=-d)

$$V_G = V(-d) = V^+ e^{+j\beta d} + \Gamma_L V^+ e^{-j\beta d}$$

$$I_G = I(-d) = \frac{V^+}{Z_o} e^{+j\beta d} - \Gamma_L \frac{V^+}{Z_o} e^{-j\beta d}$$

The rate of energy flowing through the plane at x=-d

$$P = \frac{1}{2} \mathrm{Re}\left\{ V_G I_G^{\;*} \right\}$$

$$P = \frac{1}{2} \frac{{V^+}^2}{Z_o} - \frac{1}{2} \left|\Gamma_L\right|^2 \frac{{V^+}^2}{Z_o}$$

forward power

reflected power

Incident and Reflected Power

- Power does not flow! Energy flows.
  - The forward and reflected traveling waves are power orthogonal
    - Cross terms cancel
  - The net rate of energy transfer is equal to the difference in power of the individual waves
- To maximize the power transferred to the load we want:

$$\Gamma_L = 0$$

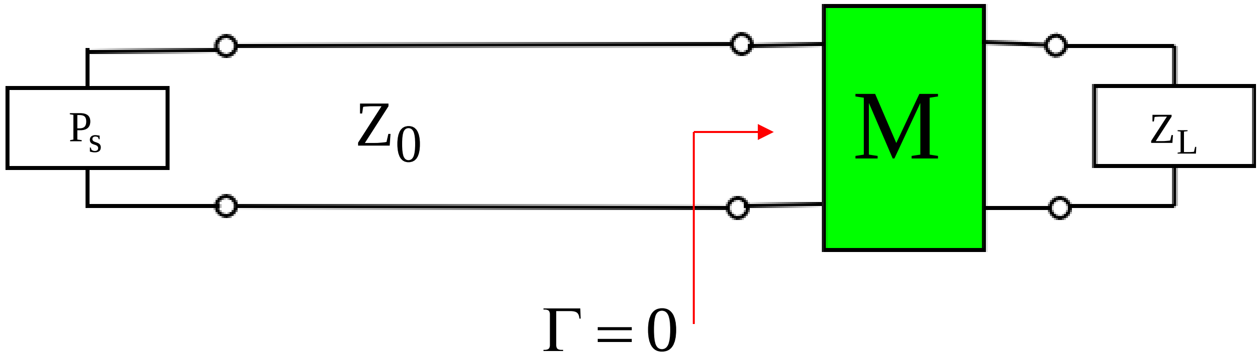
which implies:

$$Z_L = Z_o$$

When $Z_L = Z_o$, the load is matched to the transmission line

Load Matching

What if the load cannot be made equal to Z_o for some other reasons? Then, we need to build a matching network so that the source effectively sees a match load.



Typically we only want to use lossless devices such as capacitors, inductors, transmission lines, in our matching network so that we do not dissipate any power in the network and deliver all the available power to the load.

Normalized Impedance

It will be easier if we normalize the load impedance to the characteristic impedance of the transmission line attached to the load.

$$z = \frac{Z}{Z_o} = r + jx$$

$$z = \frac{1+\Gamma}{1-\Gamma}$$

Since the impedance is a complex number, the reflection coefficient will be a complex number

$$\Gamma = u + jv$$

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \qquad x = \frac{2v}{(1-u)^2+v^2}$$

Smith Charts

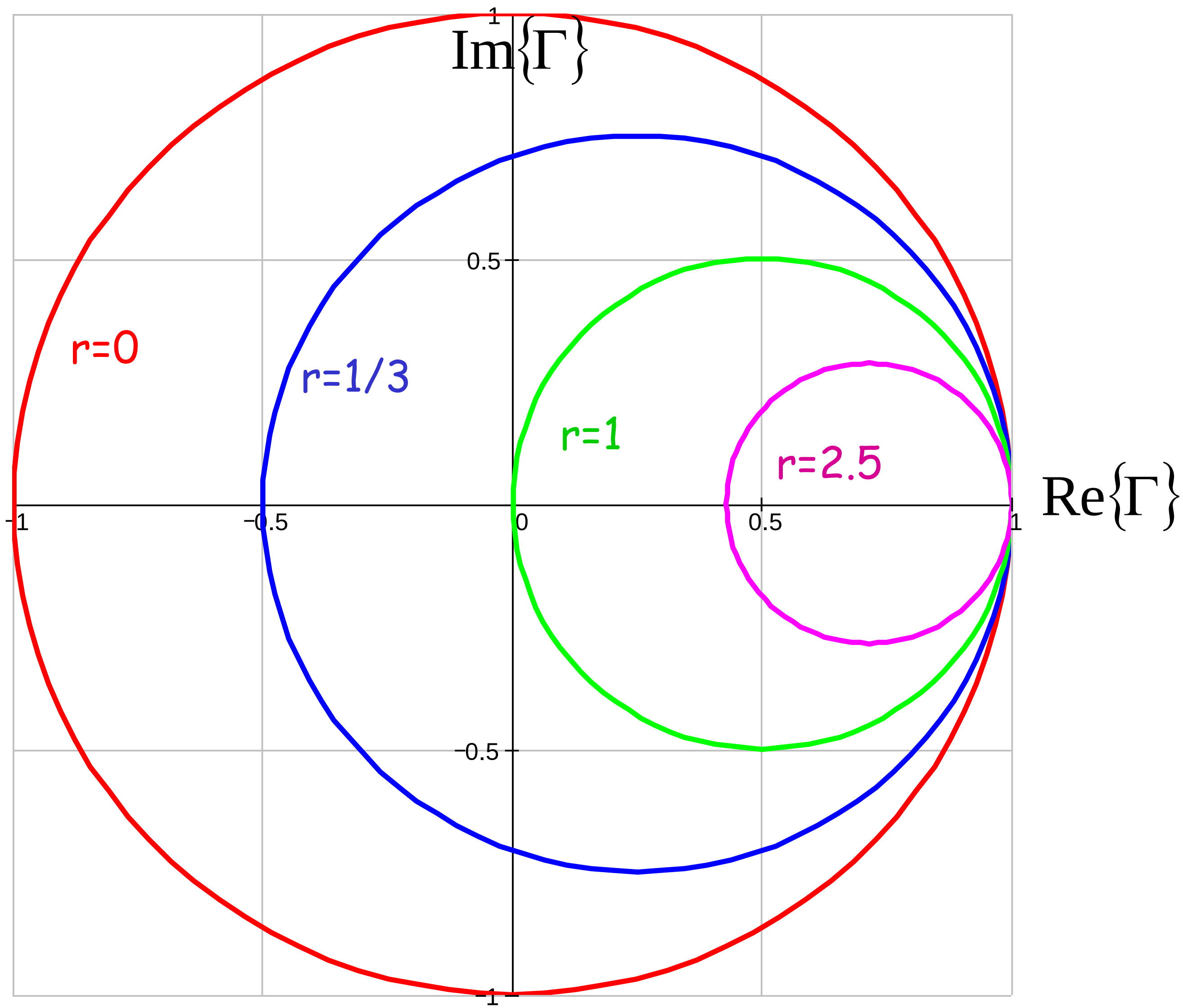
The impedance as a function of reflection coefficient can be re-written in the form:

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \quad \Rightarrow \quad \left(u-\frac{r}{1+r}\right)^2+v^2 = \frac{1}{(1+r)^2}$$

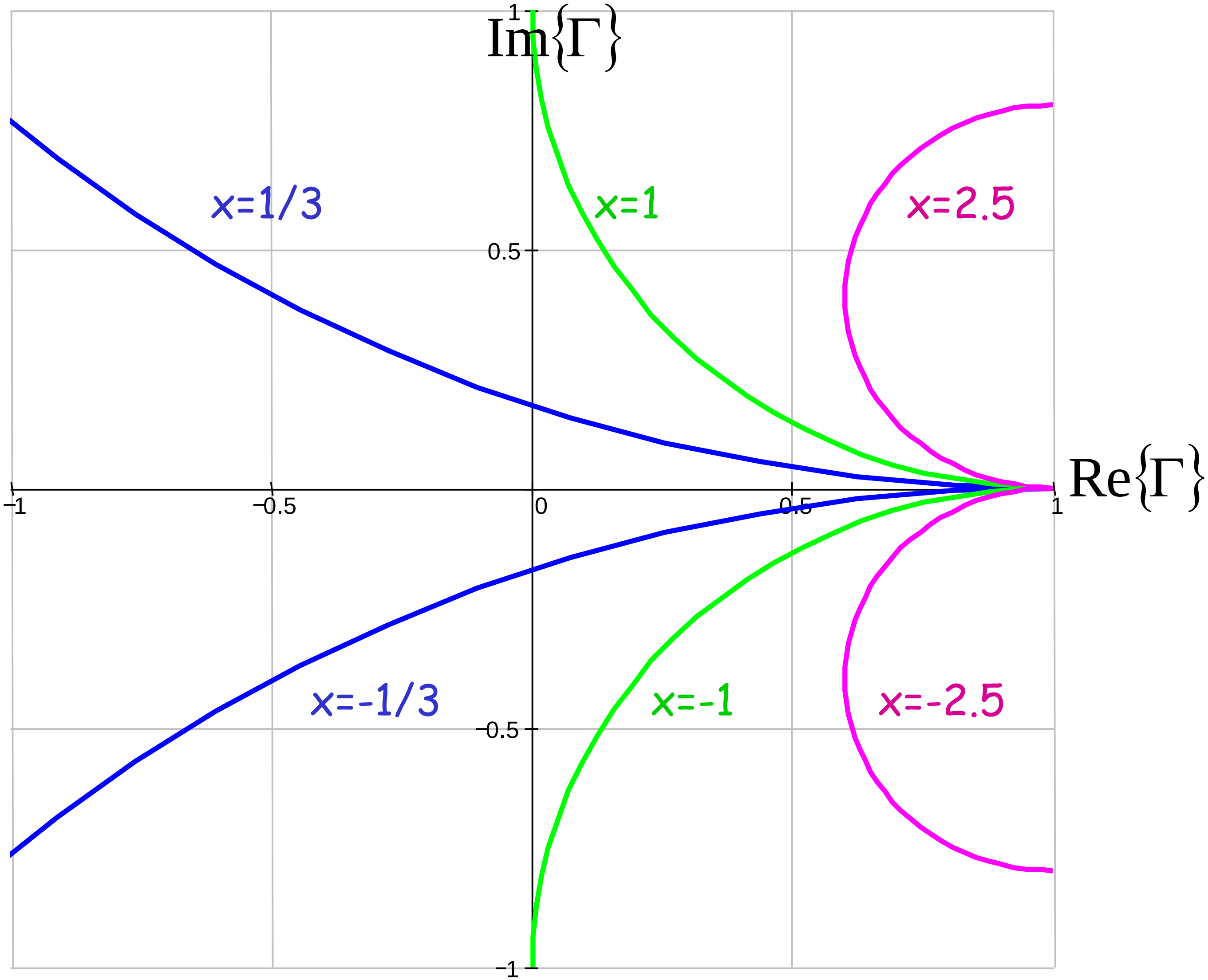
$$x = \frac{2v}{(1-u)^2+v^2} \quad \Rightarrow \quad (u-1)^2+\left(v-\frac{1}{x}\right)^2 = \frac{1}{x^2}$$

These are equations for circles on the (u,v) plane

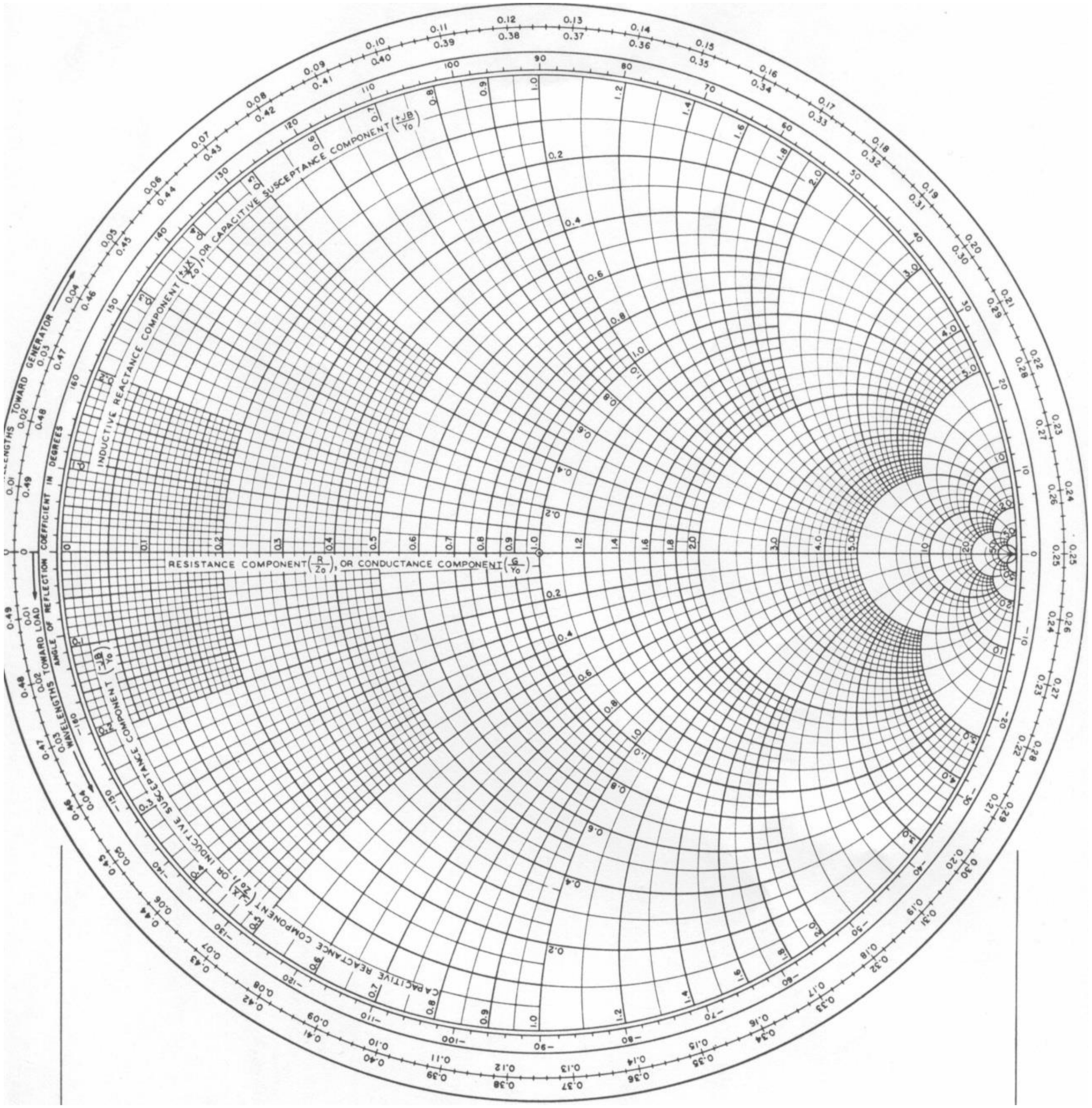
Smith Chart - Real Circles



Smith Chart - Imaginary Circles



Smith Chart


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE COMPONENT (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES

Smith Chart Example 1

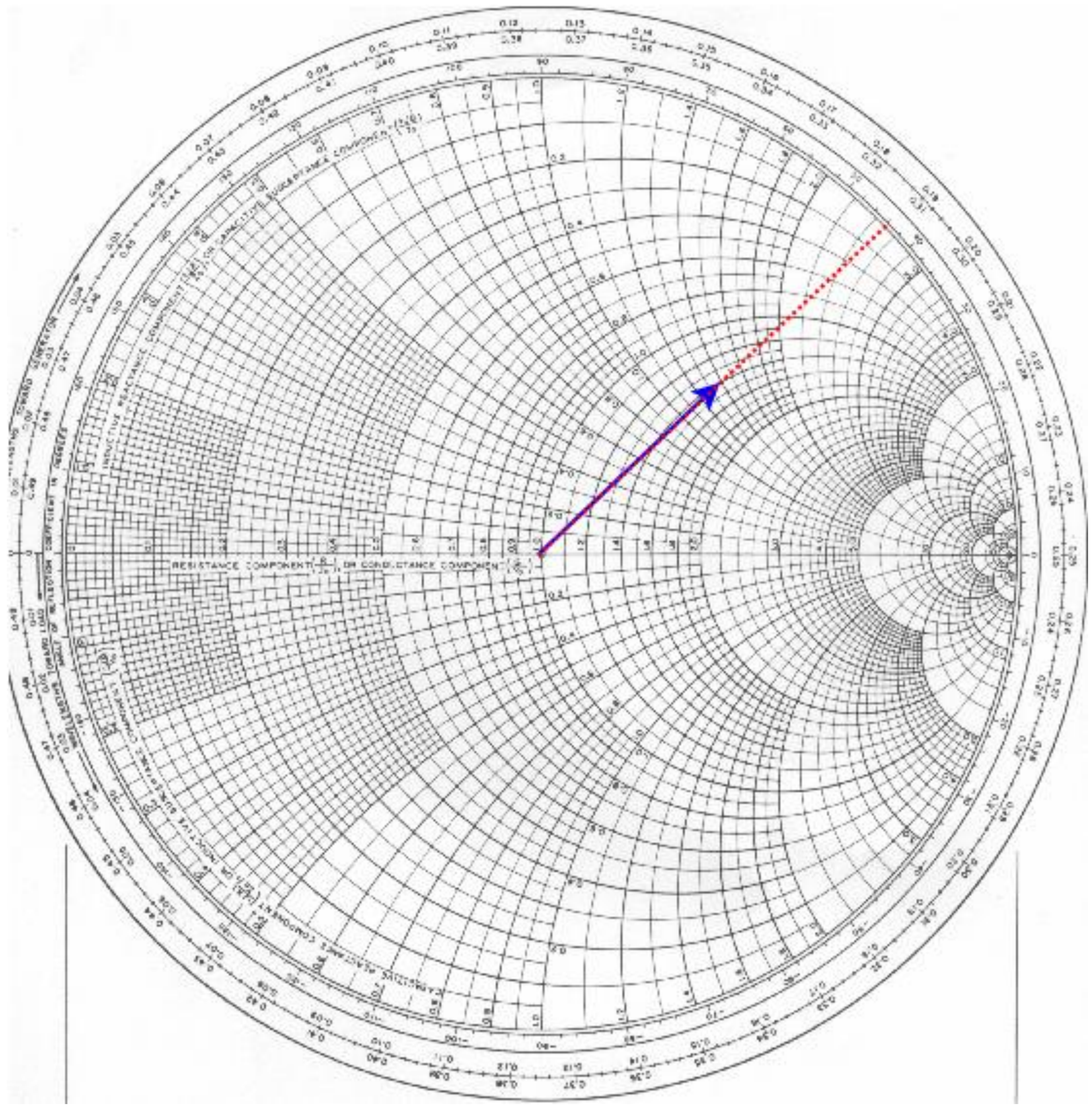
Given:

$$\Gamma_L = 0.5\angle 45°$$

$$Z_o = 50\Omega$$

What is Z_L?

$$Z_L = 50\Omega(1.35 + j1.35)$$
$$= 67.5\Omega + j67.5\Omega$$



Smith Chart Example 2

Given:

$$Z_L = 15\Omega - j25\Omega$$

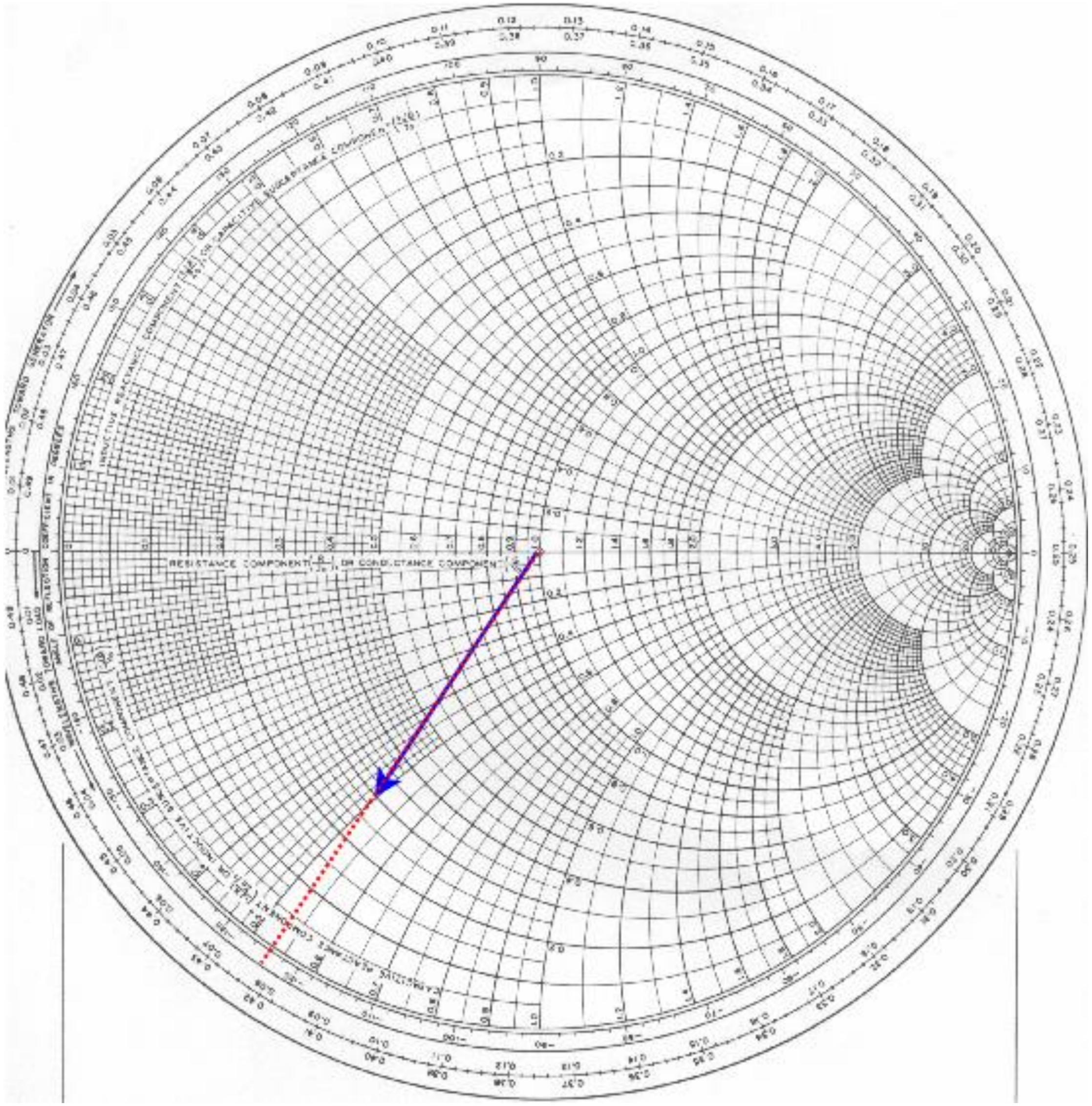
$$Z_o = 50\Omega$$

What is Γ_L?

$$z_L = \frac{15\Omega - j25\Omega}{50\Omega}$$

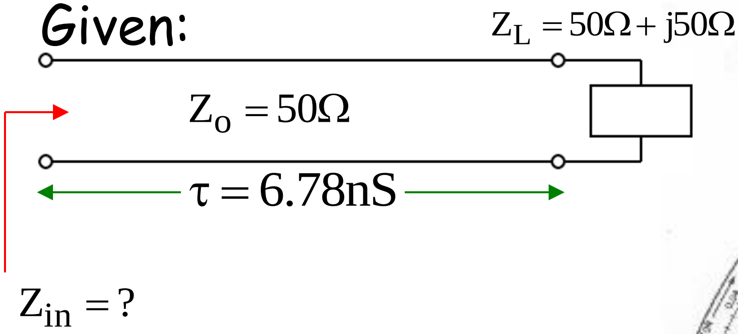
$$= 0.3 - j0.5$$

$$\Gamma_L = 0.618\angle -124°$$



Smith Chart Example 3

Given:



What is Z_{in} at 50 MHz?

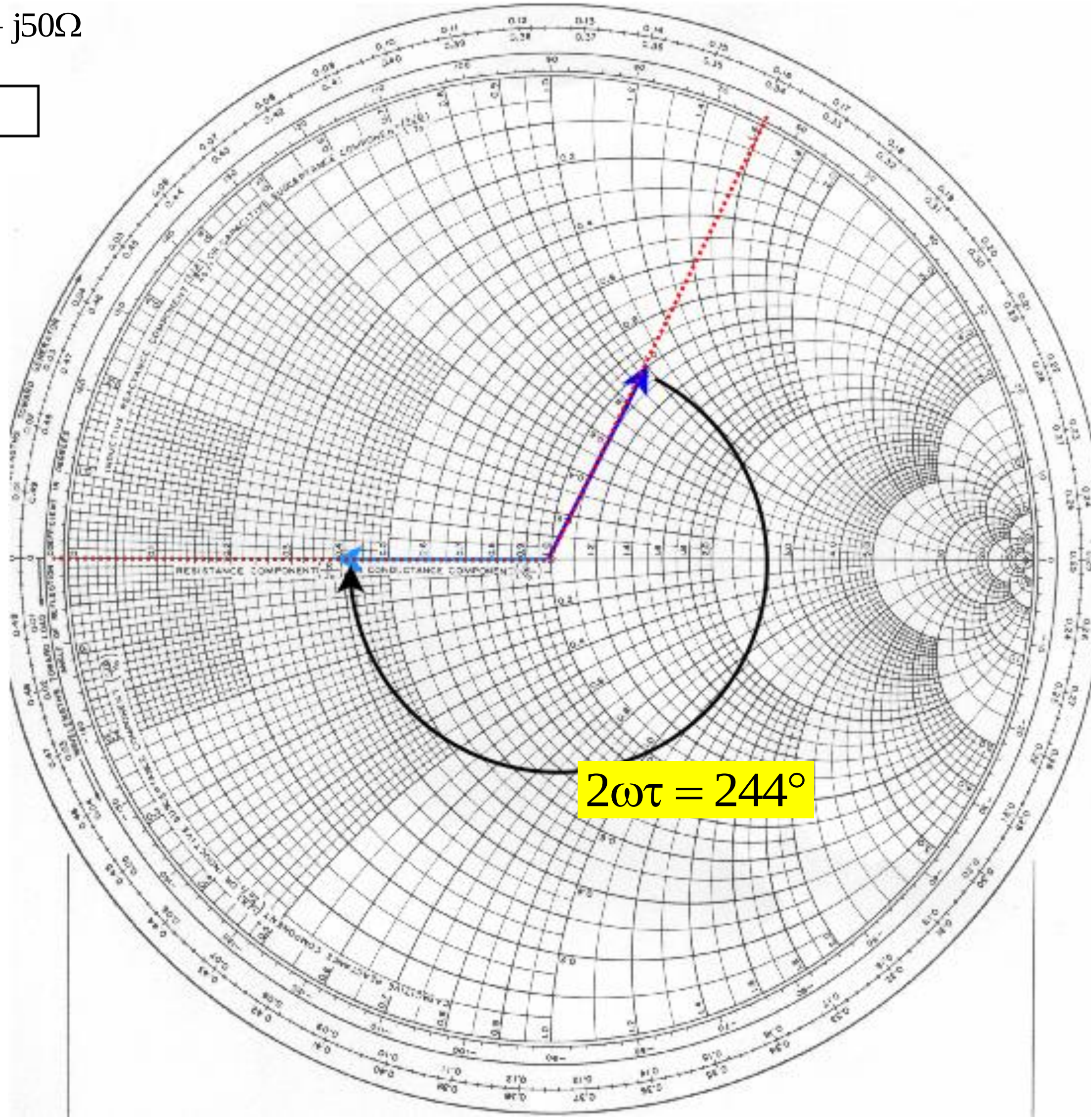
$$z_L = \frac{50\Omega + j50\Omega}{50\Omega}$$
$$= 1.0 + j1.0$$

$$\Gamma_L = 0.445\angle 64°$$

$$\Gamma_{in} = \Gamma_L e^{-j2\beta d} = \Gamma_L e^{-j2\omega\tau}$$

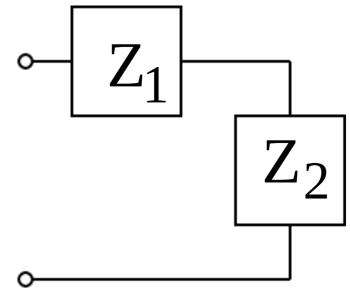
$$2\omega\tau = 244°$$
$$\Gamma_{in} = 0.445\angle 180°$$

$$Z_{in} = 50\Omega(0.38 + j0.0) = 19\Omega$$



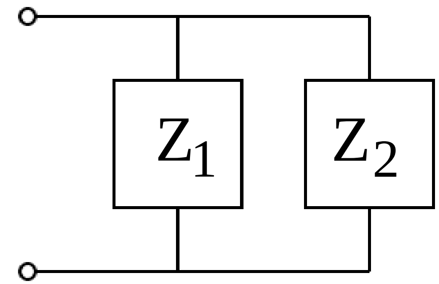
Admittance

A matching network is going to be a combination of elements connected in series AND parallel.



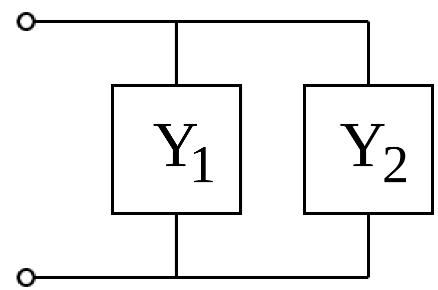
Impedance is well suited when working with series configurations. For example:

$$V = ZI \qquad\qquad Z_L = Z_1 + Z_2$$



Impedance is NOT well suited when working with parallel configurations.

$$Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$$



For parallel loads it is better to work with admittance.

$$I = YV \qquad Y_1 = \frac{1}{Z_1} \qquad Y_L = Y_1 + Y_2$$

Normalized Admittance

$$y = \frac{Y}{Y_o} = YZ_o = g + jb$$

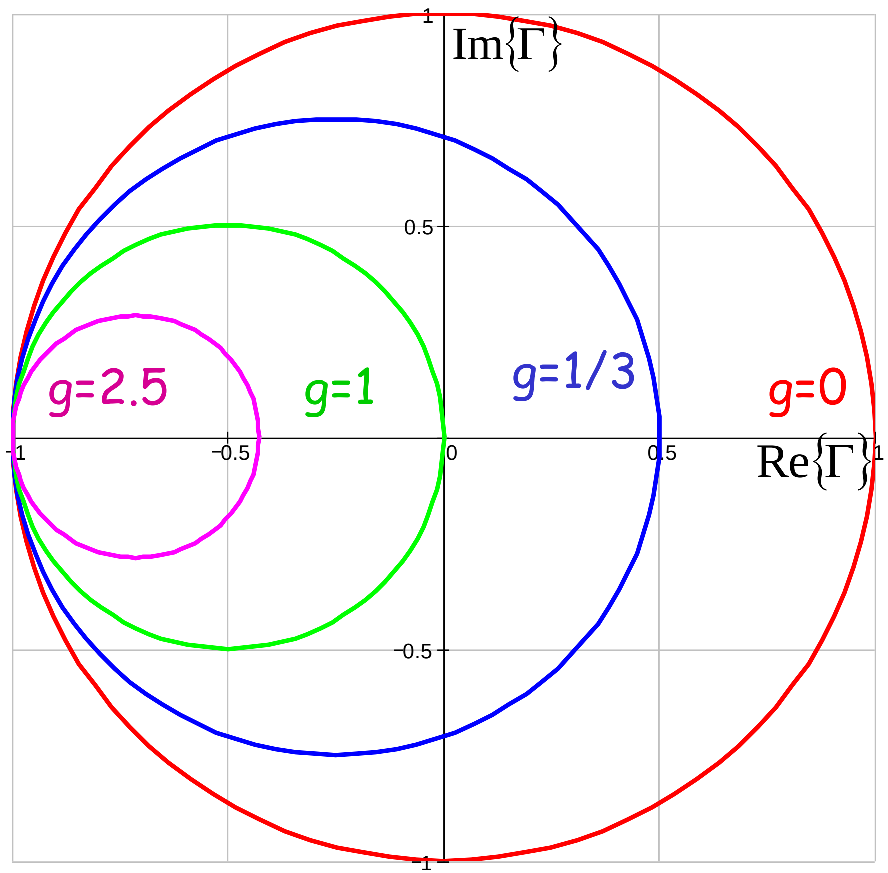
$$y = \frac{1-\Gamma}{1+\Gamma}$$

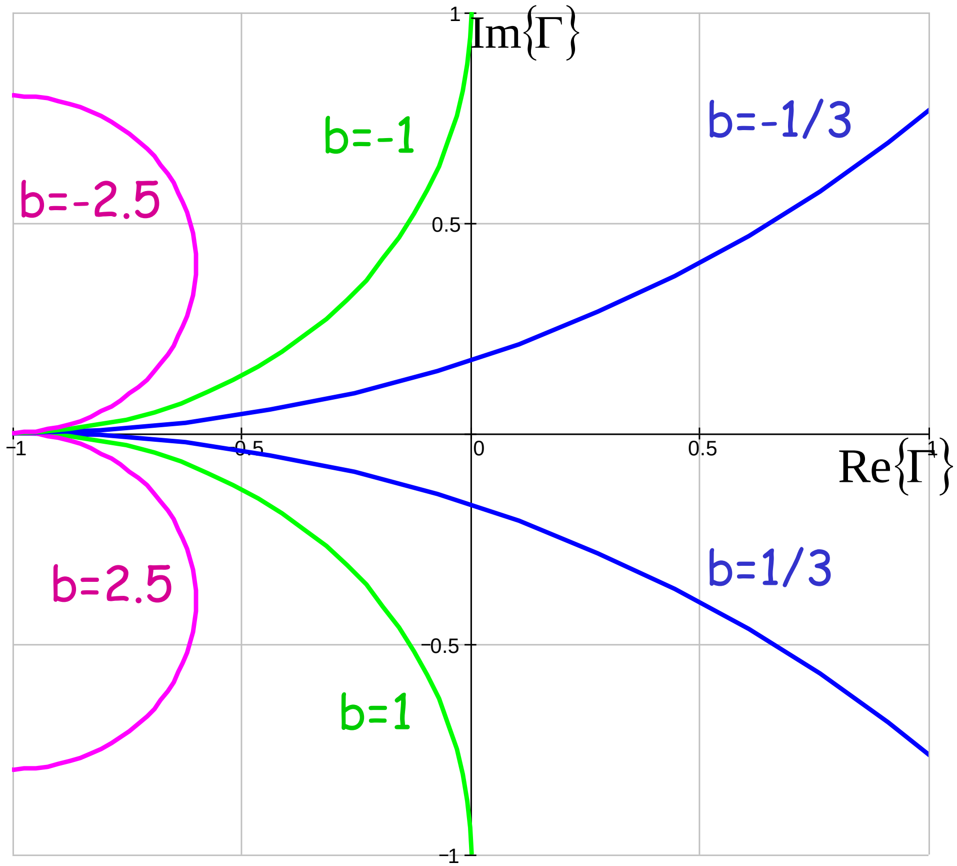
$$g = \frac{1-u^2-v^2}{(1+u)^2+v^2} \quad \Rightarrow \quad \left(u+\frac{g}{1+g}\right)^2 + v^2 = \frac{1}{(1+g)^2}$$

$$b = \frac{-2v}{(1+u)^2+v^2} \quad \Rightarrow \quad (u+1)^2 + \left(v+\frac{1}{b}\right)^2 = \frac{1}{b^2}$$

These are equations for circles on the (u,v) plane

Admittance Smith Chart





Impedance and Admittance Smith Charts

- For a matching network that contains elements connected in series and parallel, we will need two types of Smith charts
  - impedance Smith chart
  - admittance Smith Chart
- The admittance Smith chart is the impedance Smith chart rotated 180 degrees.
  - We could use one Smith chart and flip the reflection coefficient vector 180 degrees when switching between a series configuration to a parallel configuration.

Admittance Smith Chart Example 1

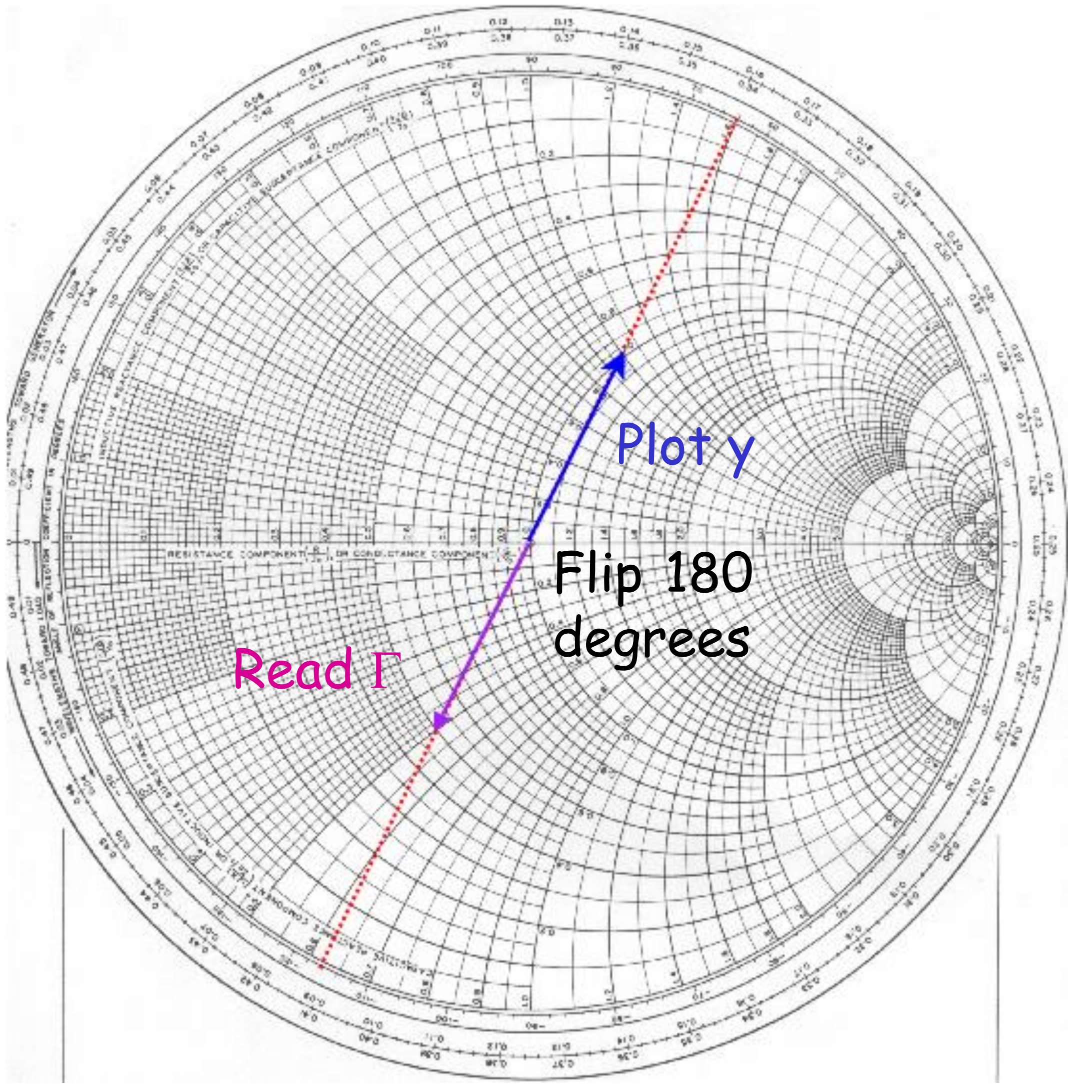
Given:

$$y = 1 + j1$$

What is Γ?

- Procedure:
  - Plot 1+j1 on chart
    - vector = $0.445\angle 64°$
  - Flip vector 180 degrees

$$\Gamma = 0.445\angle -116°$$



Admittance Smith Chart Example 2

Given:

$$\Gamma = 0.5\angle +45^\circ \quad Z_o = 50\Omega$$

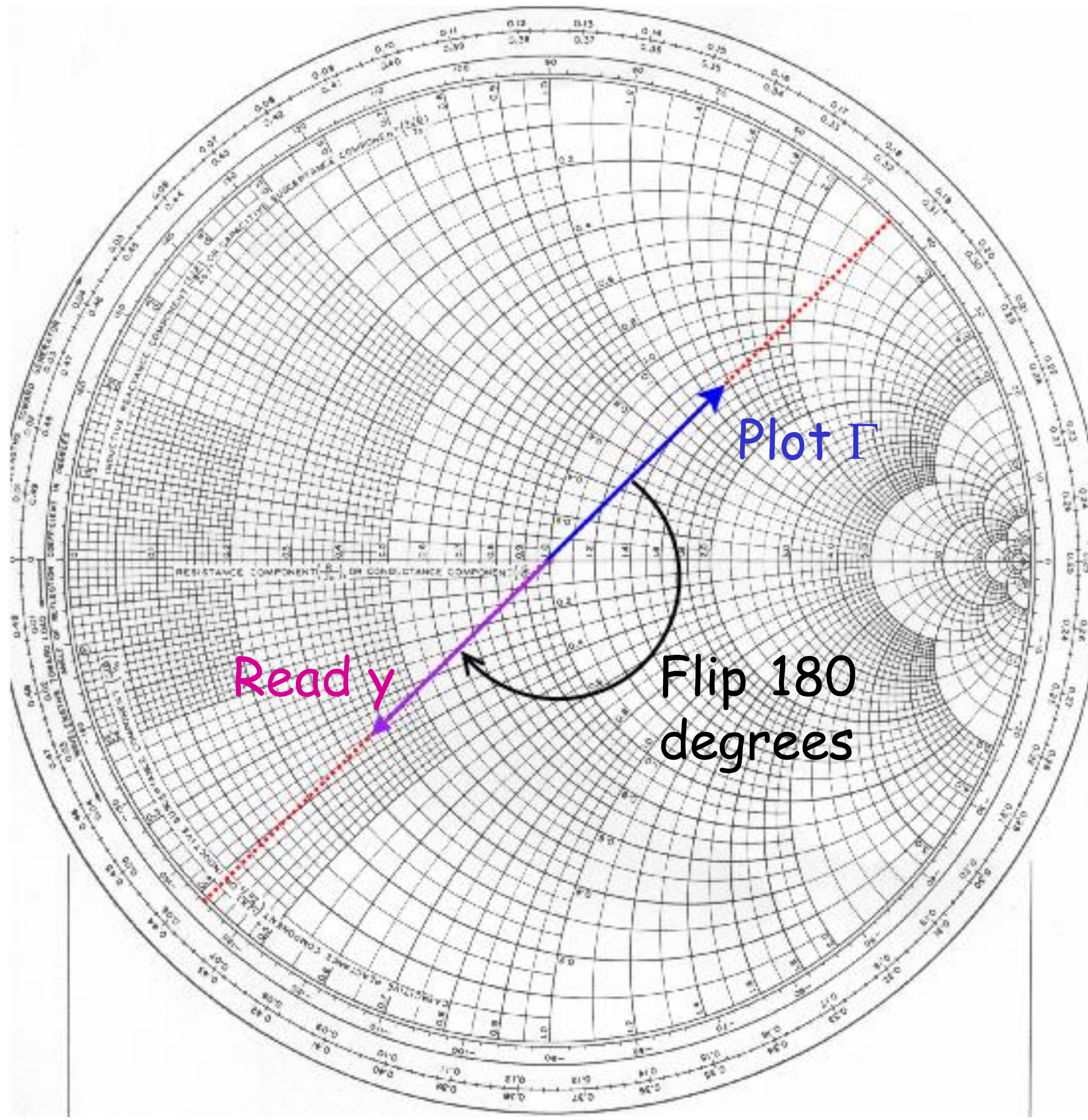
What is Y?

- Procedure:
  - Plot Γ
  - Flip vector by 180 degrees
  - Read coordinate

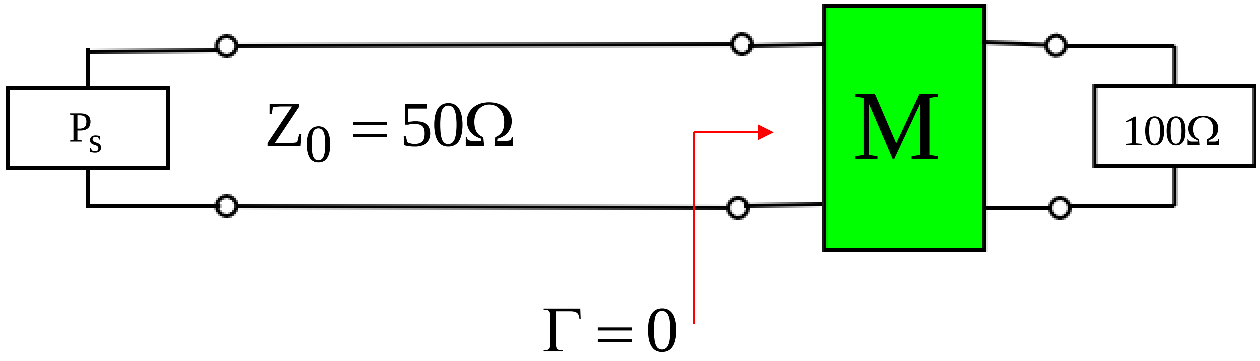
$$y = 0.38 - j0.36$$

$$Y = \frac{1}{50\Omega}\left(0.38 - j0.36\right)$$

$$Y = \left(7.6 - j7.2\right)x10^{-3}\text{mhos}$$



Matching Example

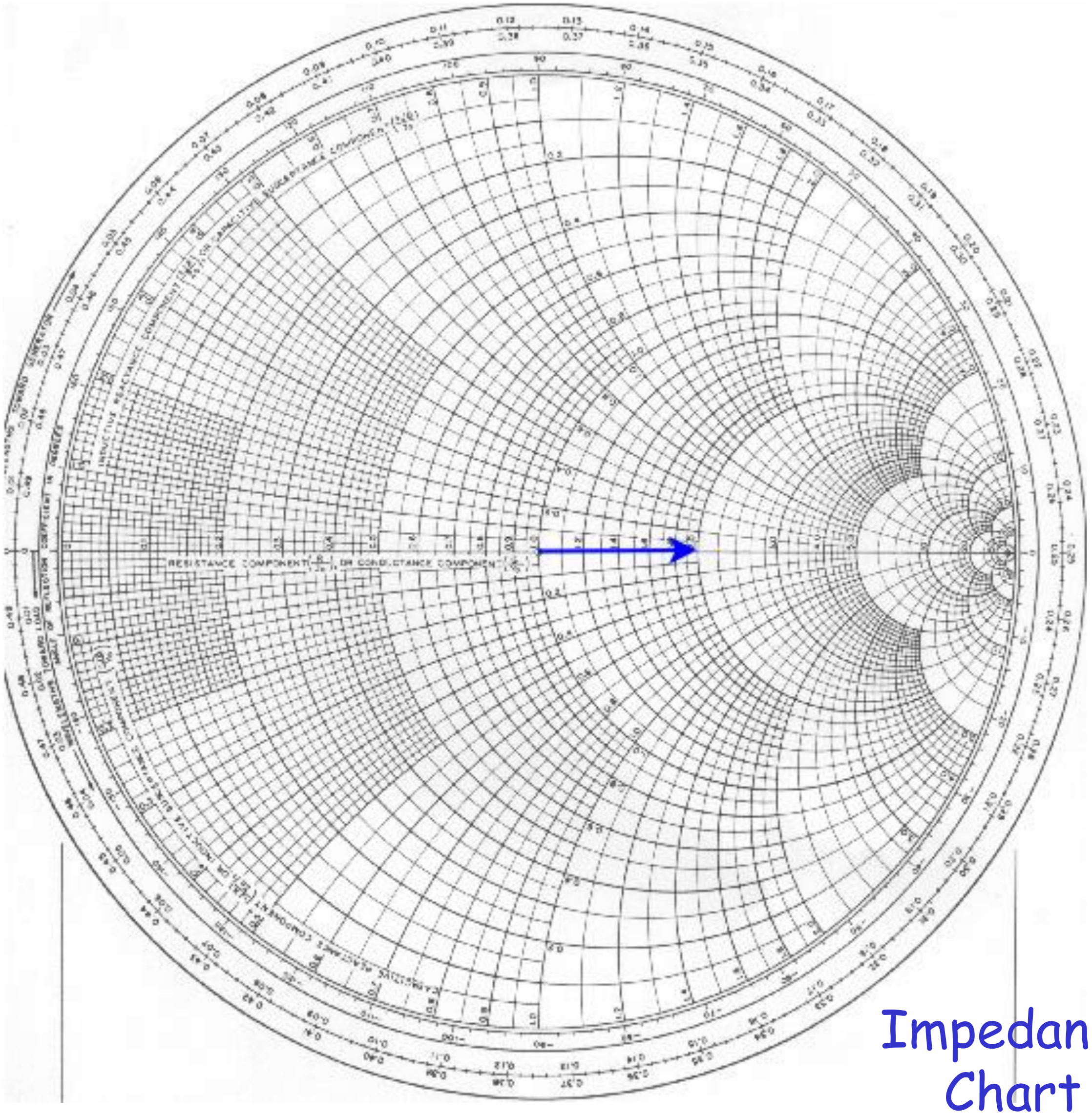


Match 100Ω load to a 50Ω system at 100MHz

A 100Ω resistor in parallel would do the trick but ½ of the power would be dissipated in the matching network. We want to use only lossless elements such as inductors and capacitors so we don't dissipate any power in the matching network

Matching Example

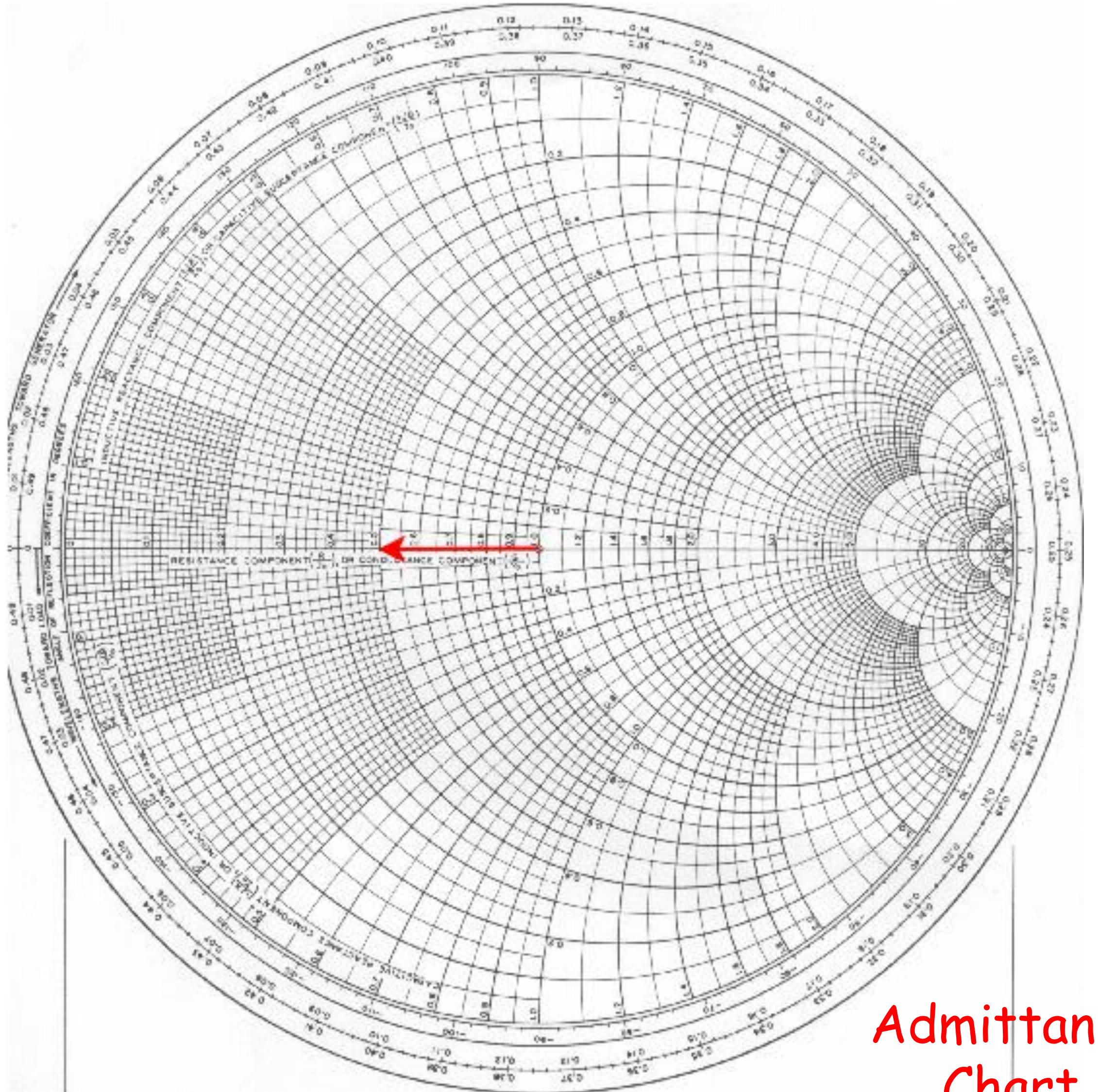
- We need to go from z=2+j0 to z=1+j0 on the Smith chart
- We won't get any closer by adding series impedance so we will need to add something in parallel.
- We need to flip over to the admittance chart



Impedance Chart

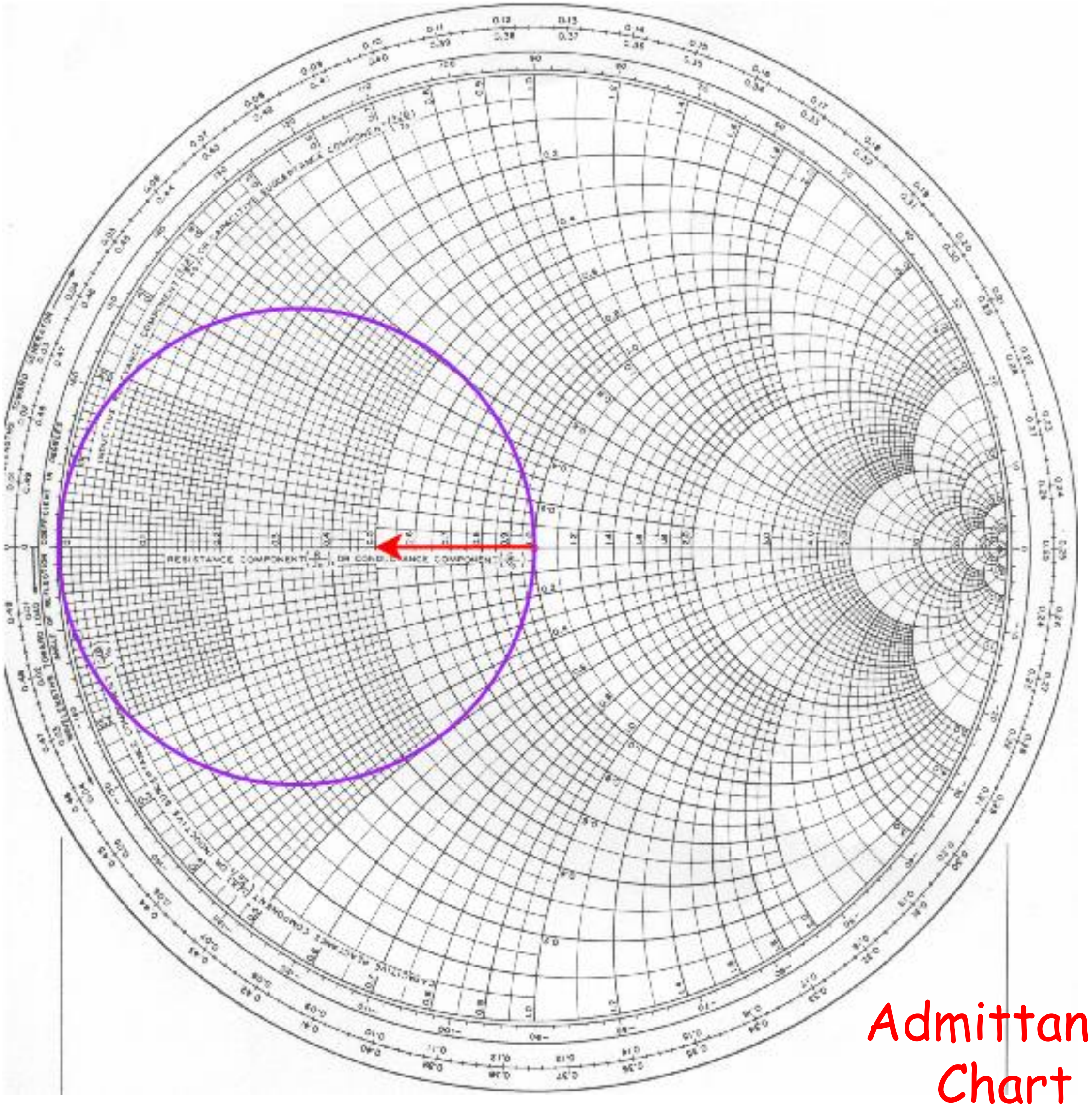
Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide.



Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance.



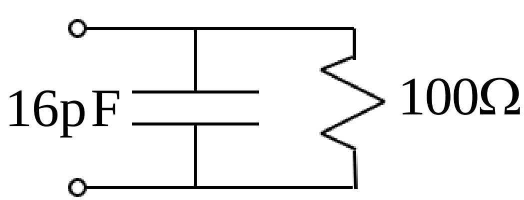
Matching Example

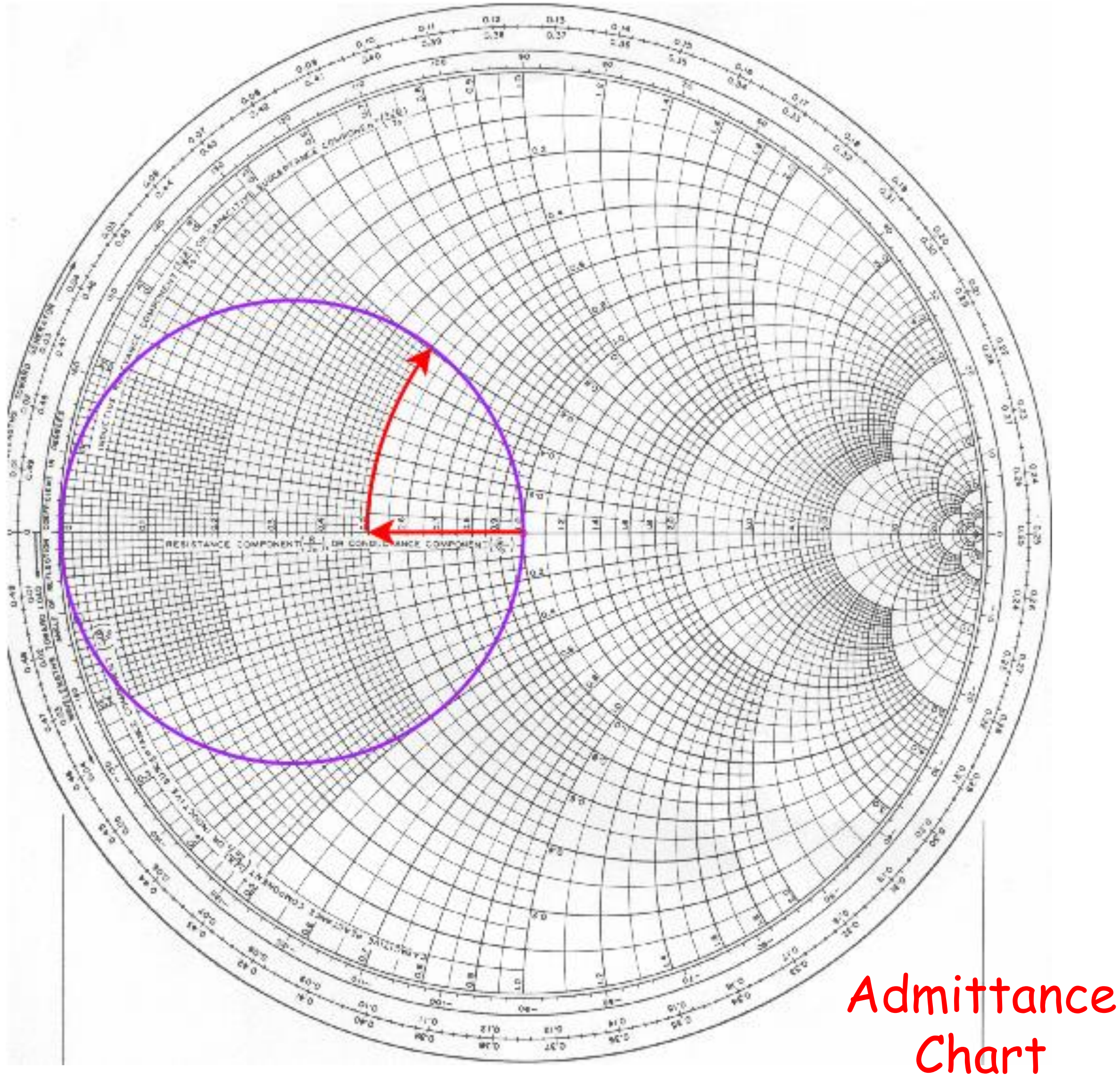
- $y=0.5+j0$
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance jb = j0.5

$$jb = j0.5$$

$$\frac{j0.5}{50\Omega} = j2\pi(100\,\mathrm{MHz})C$$

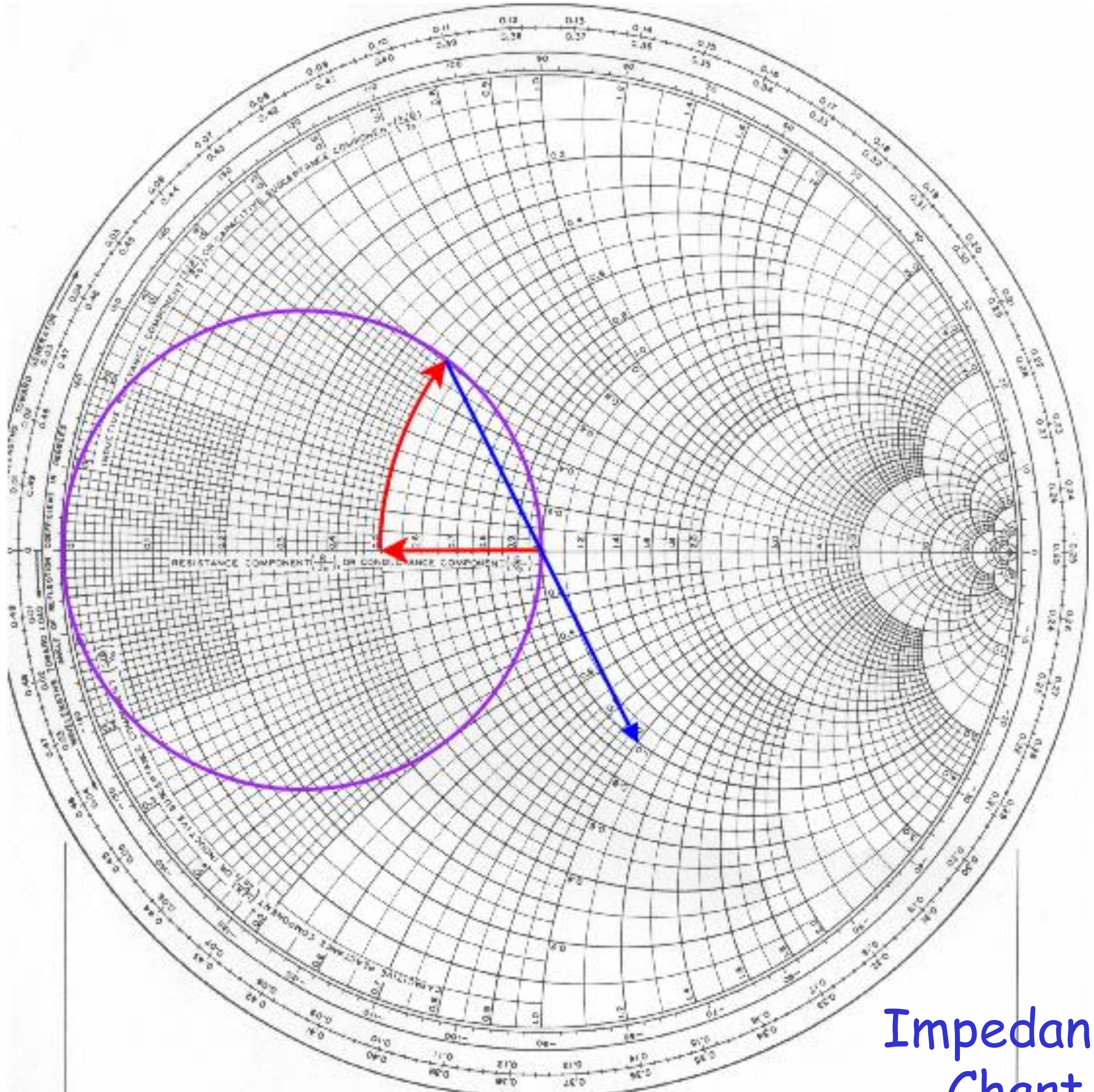
$$C = 16\,\mathrm{pF}$$





Matching Example

- We will now add series impedance
- Flip to the impedance Smith Chart
- We land at on the r=1 circle at x=-1



Impedance Chart

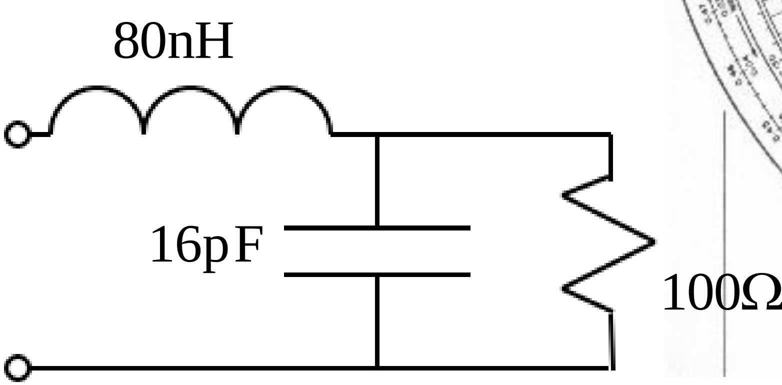
Matching Example

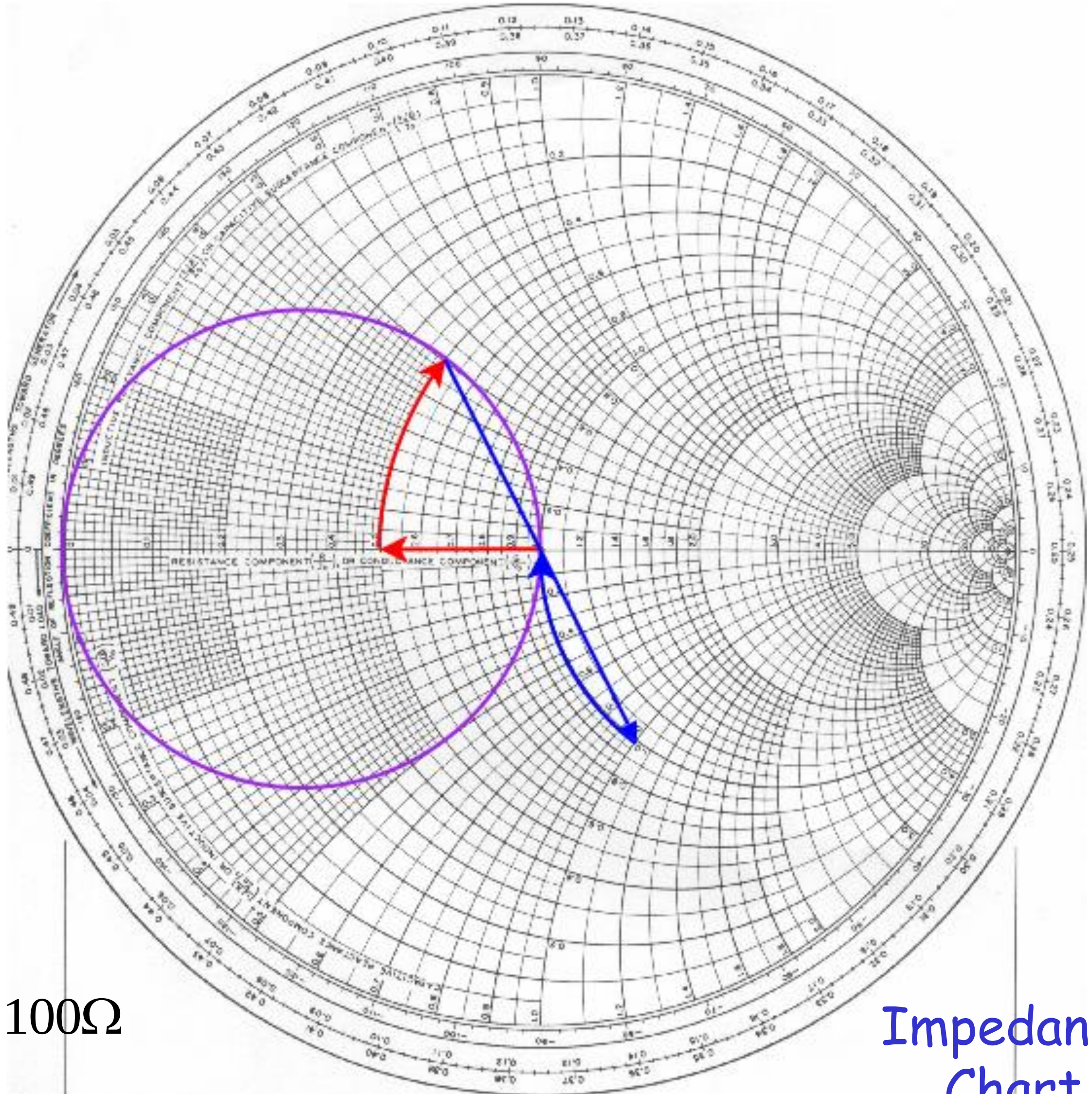
- Add positive imaginary admittance to get to z=1+j0

$$jx = j1.0$$

$$(j1.0)50\Omega = j2\pi(100\,\mathrm{MHz})L$$

$$L = 80\,\mathrm{nH}$$

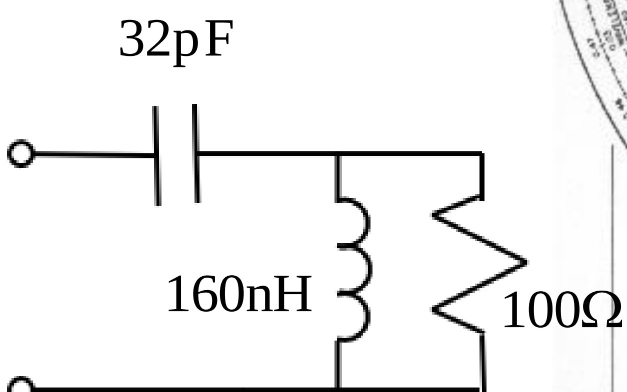


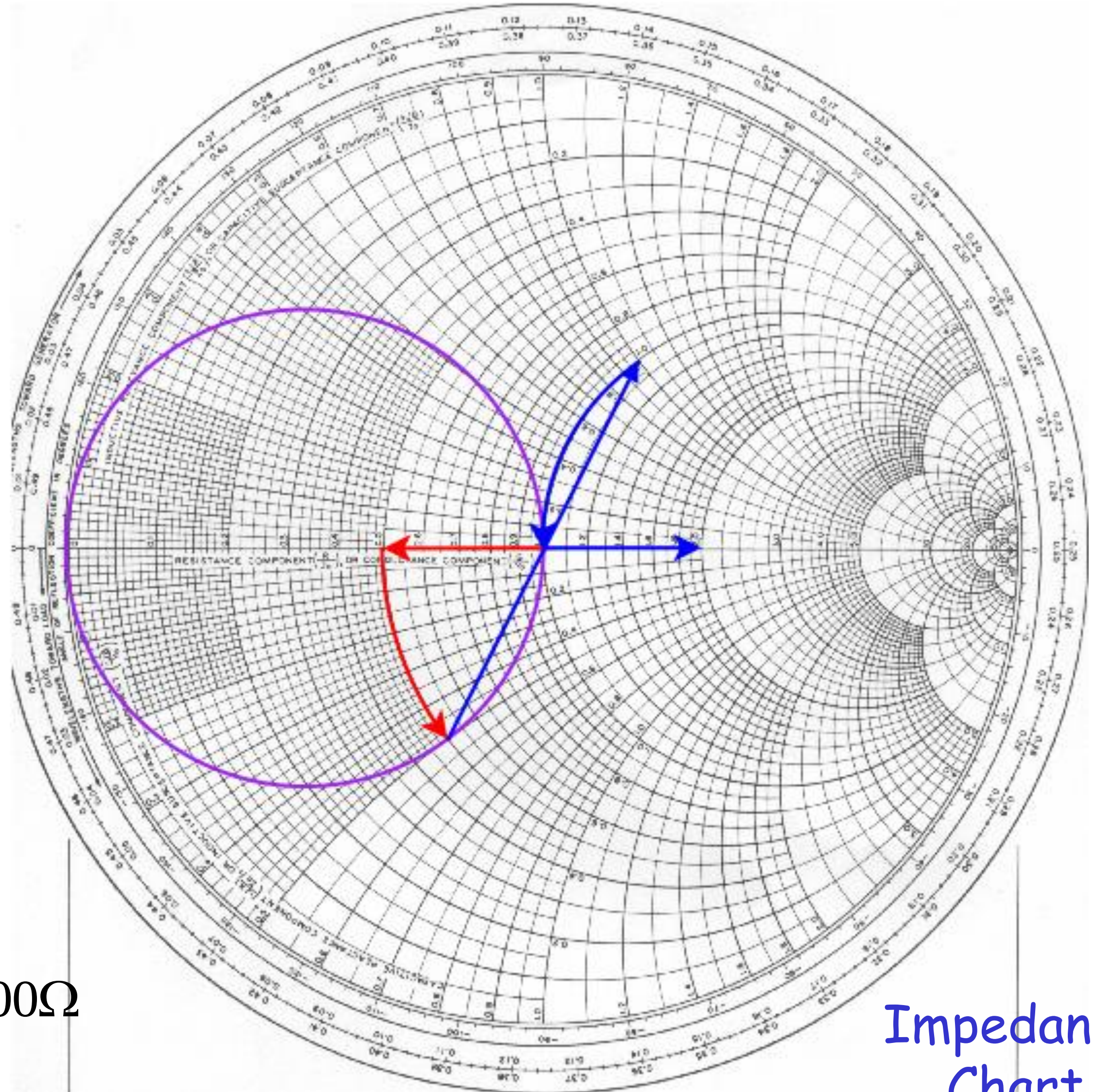


Impedance Chart

Matching Example

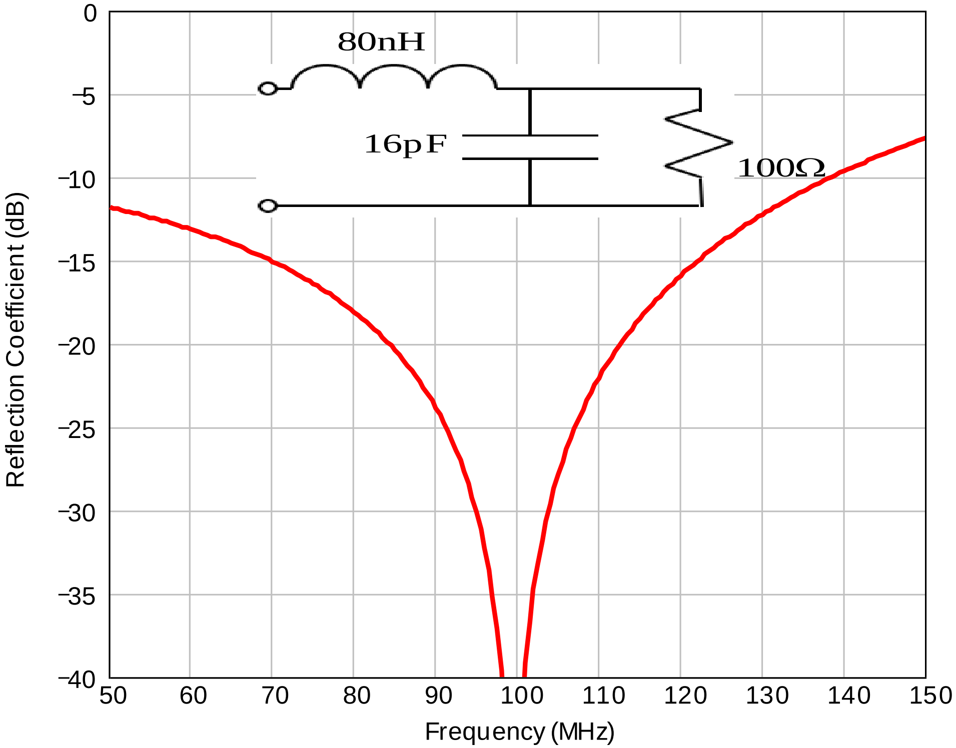
- This solution would have also worked

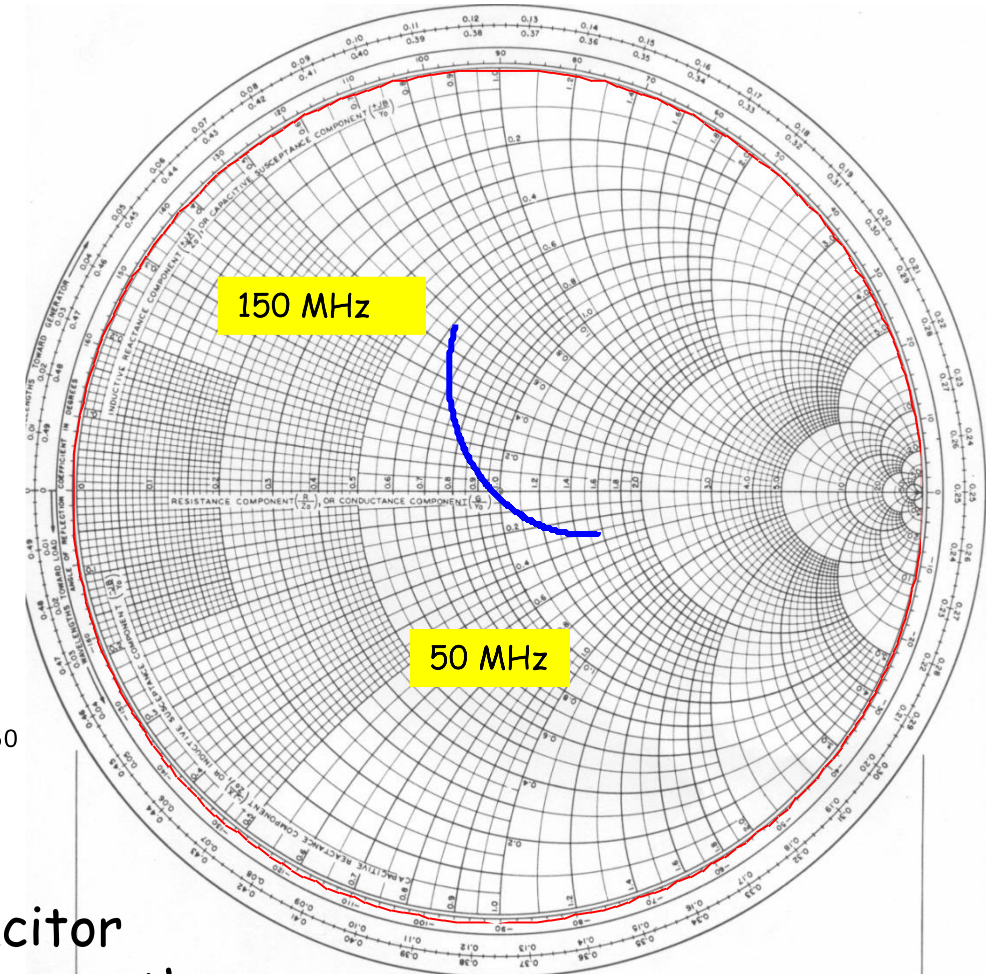




Impedance Chart

Matching Bandwidth





Because the inductor and capacitor impedances change with frequency, the match works over a narrow frequency range

Impedance Chart

dB and dBm

A dB is defined as a **POWER** ratio. For example:

$$\Gamma_{dB} = 10\log\left(\frac{P_{rev}}{P_{for}}\right)$$

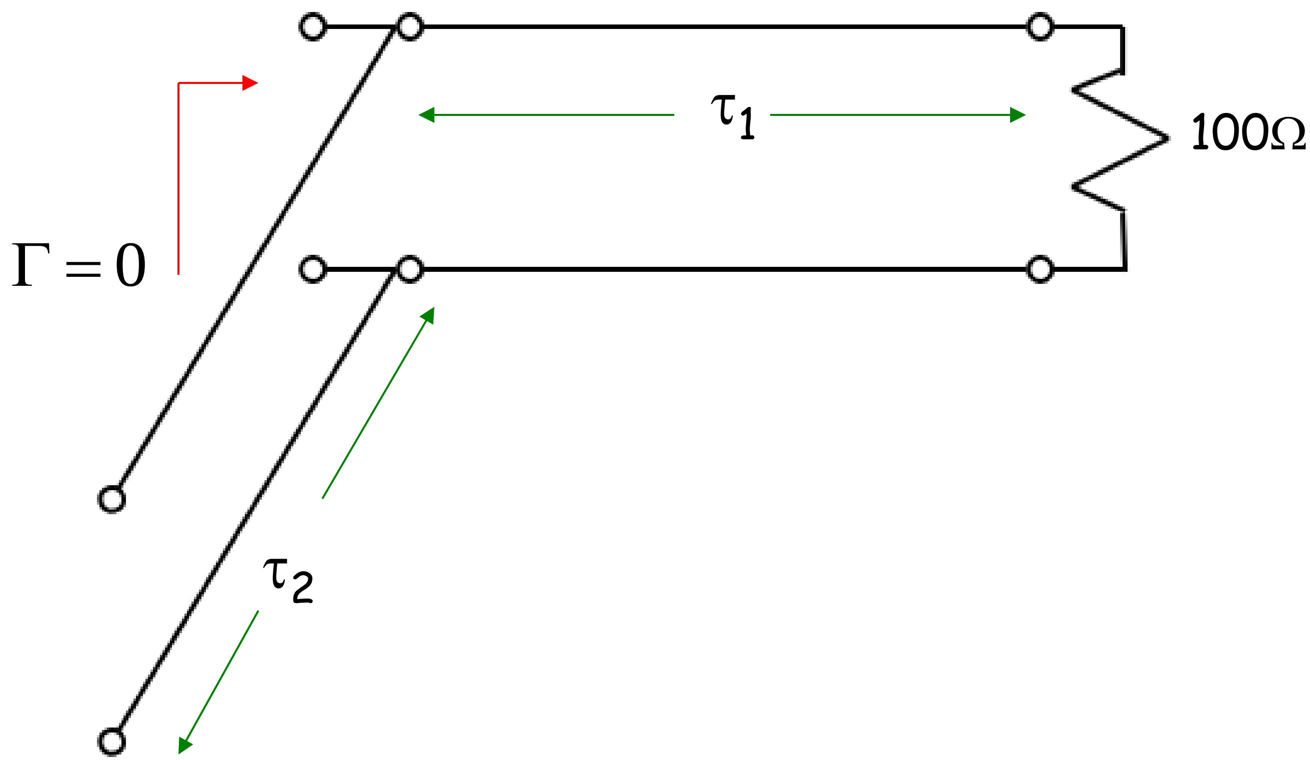
$$= 10\log\left(|\Gamma|^2\right)$$

$$= 20\log\left(|\Gamma|\right)$$

A dBm is defined as log unit of power referenced to 1mW:

$$P_{dBm} = 10\log\left(\frac{P}{1mW}\right)$$

Single Stub Tuner



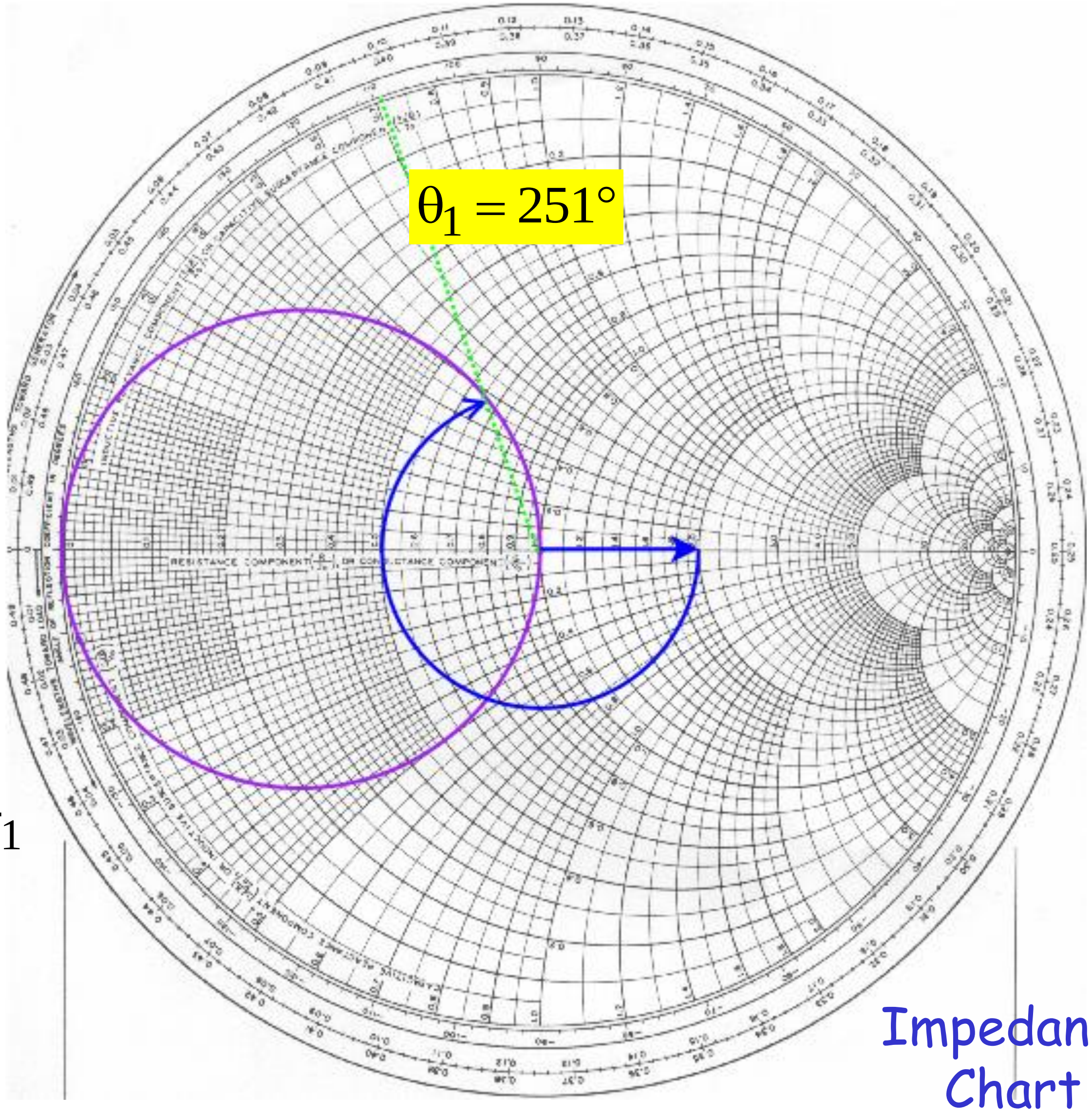
Match 100Ω load to a 50Ω system at 100MHz using two transmission lines connected in parallel

Single Stub Tuner

- Adding length to Cable 1 rotates the reflection coefficient clockwise.
- Enough cable is added so that the reflection coefficient reaches the mirror image circle

$$251° = 2 \times 360° \times 100\text{MHz} \times \tau_1$$

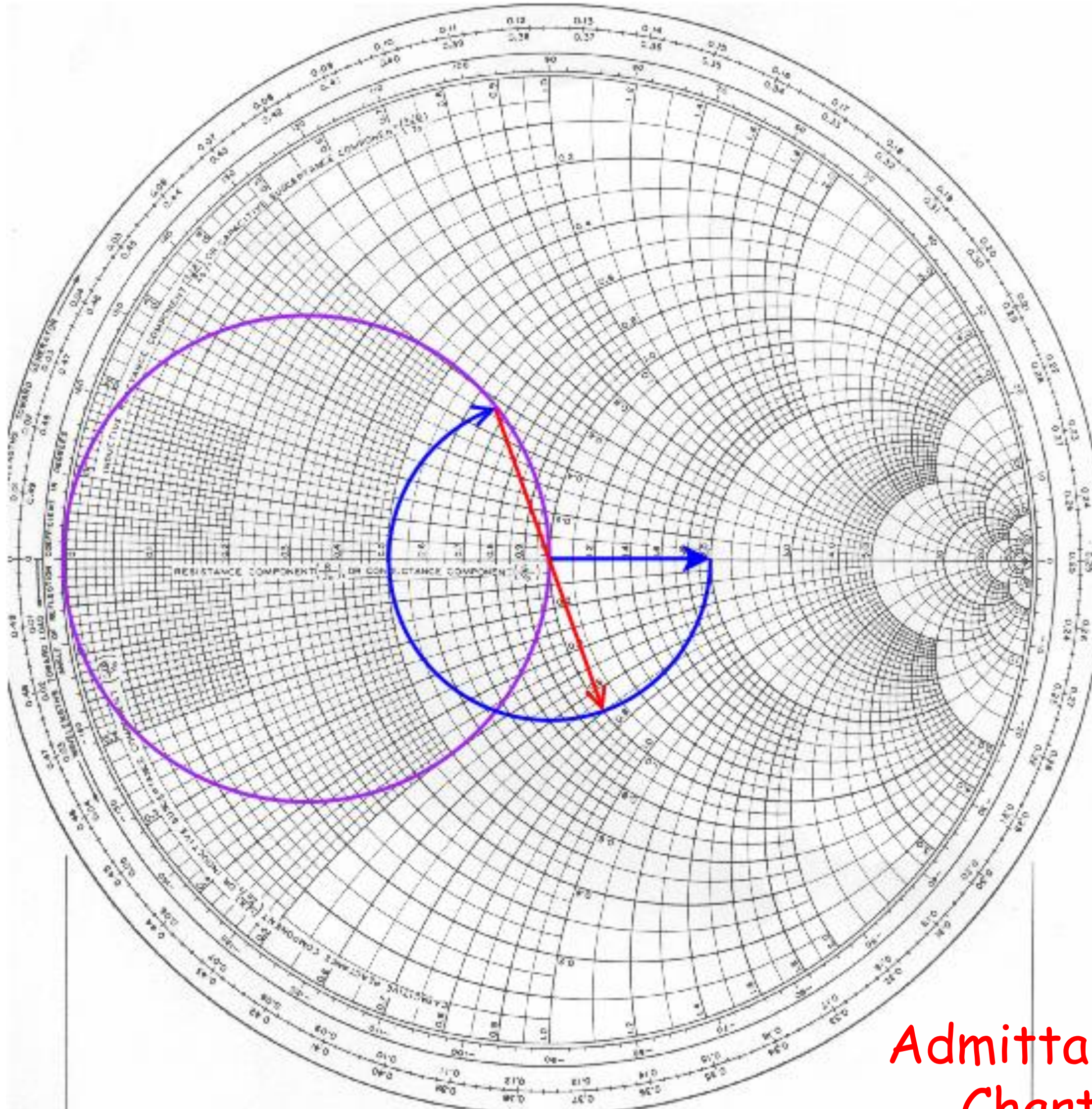
$$\tau_1 = 3.49\text{nS}$$



Impedance Chart

Single Stub Tuner

- The stub is going to be added in parallel so flip to the admittance chart.
- The stub has to add a normalized admittance of 0.7 to bring the trajectory to the center of the Smith Chart



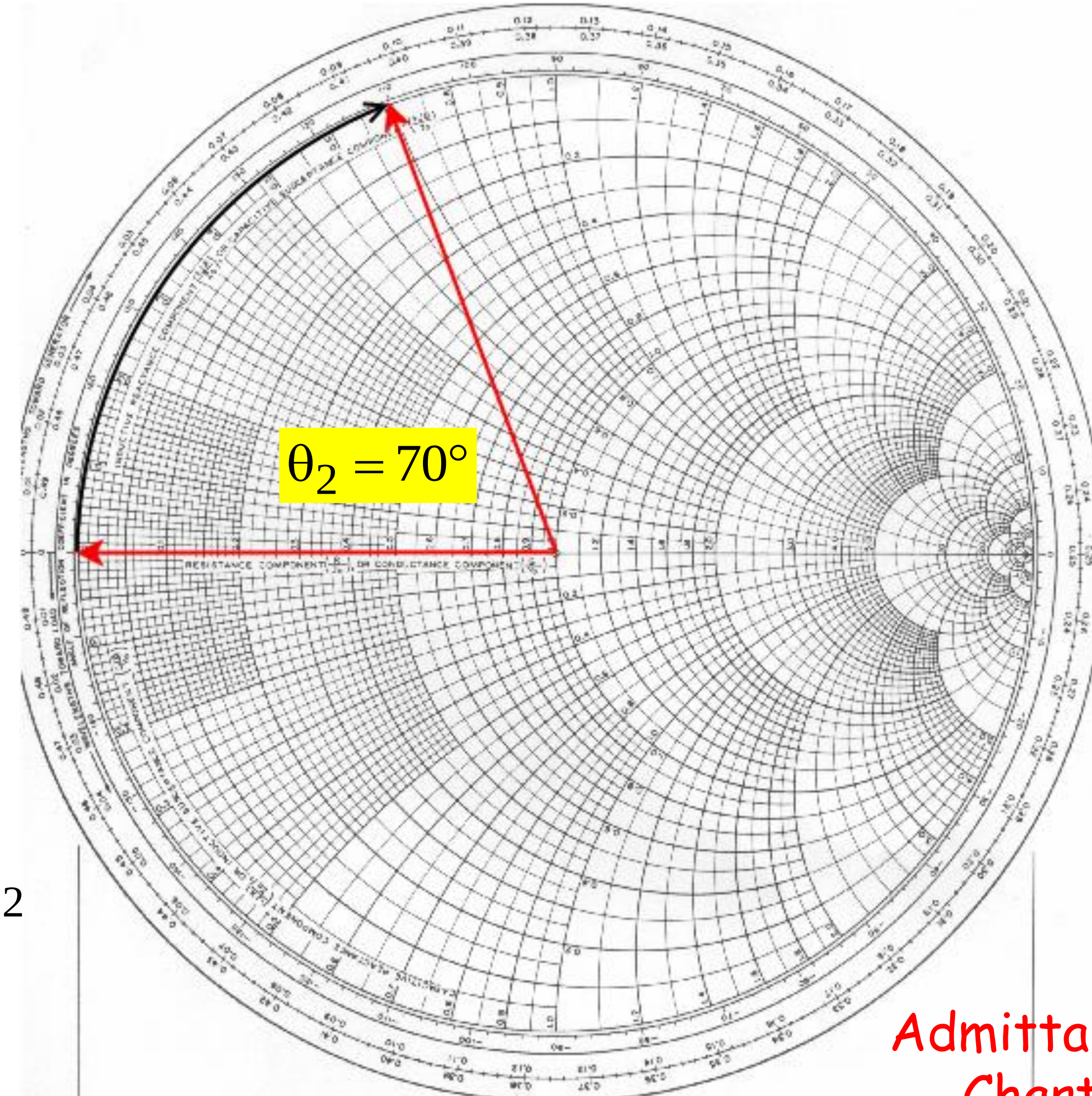
Admittance Chart

Single Stub Tuner

- An open stub of zero length has an admittance=j0.0
- By adding enough cable to the open stub, the admittance of the stub will increase.
- 70 degrees will give the open stub an admittance of j0.7

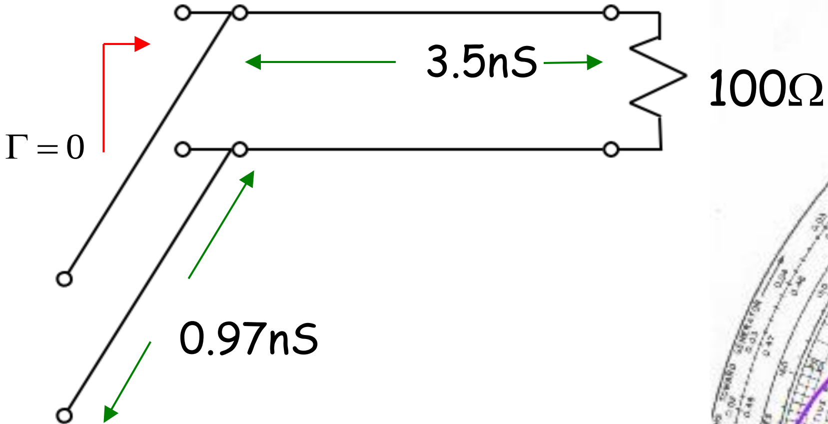
$$70^\circ = 2 \times 360^\circ \times 100\,\mathrm{MHz} \times \tau_2$$

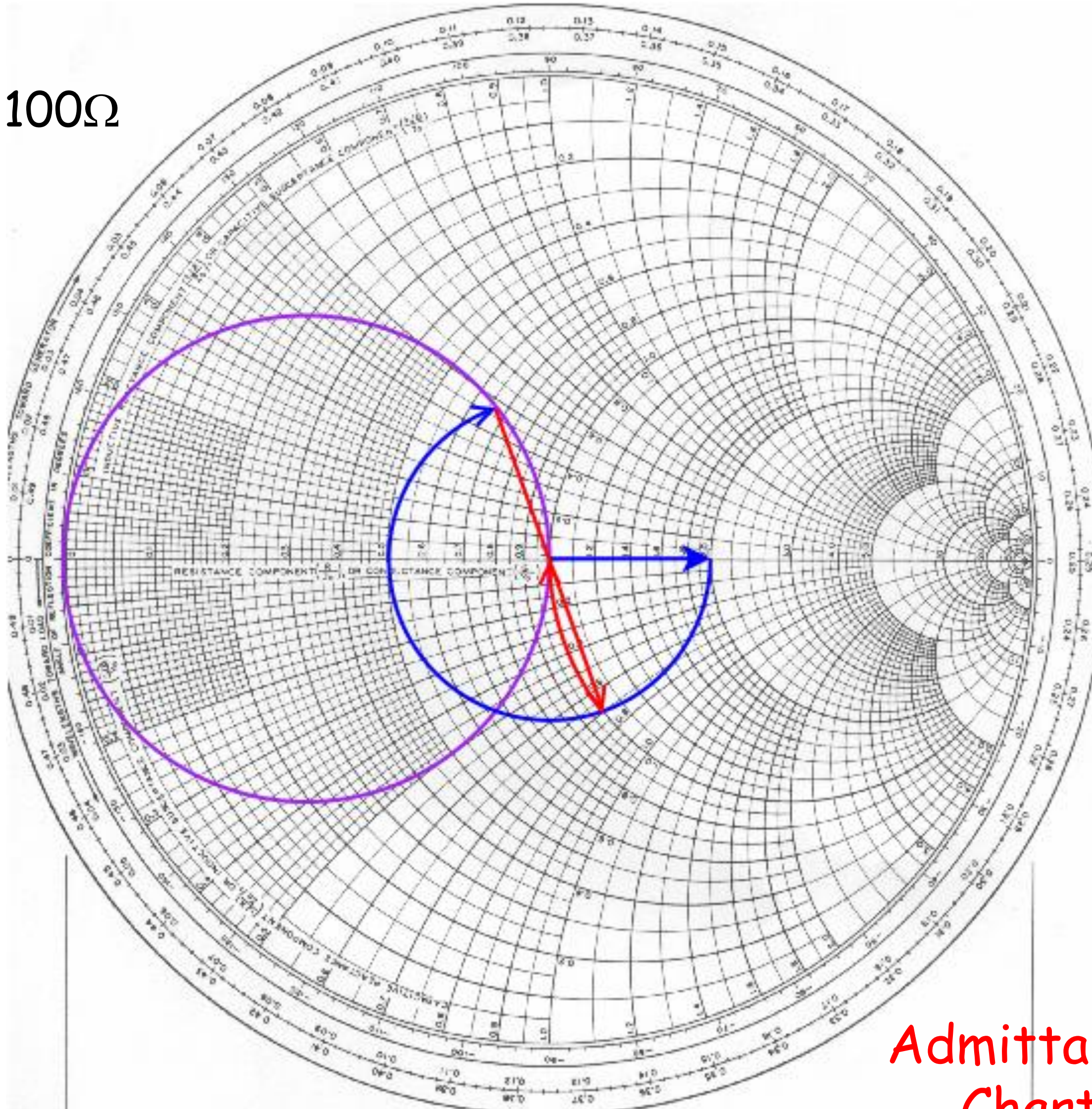
$$\tau_2 = 0.97\,\mathrm{nS}$$



Admittance Chart

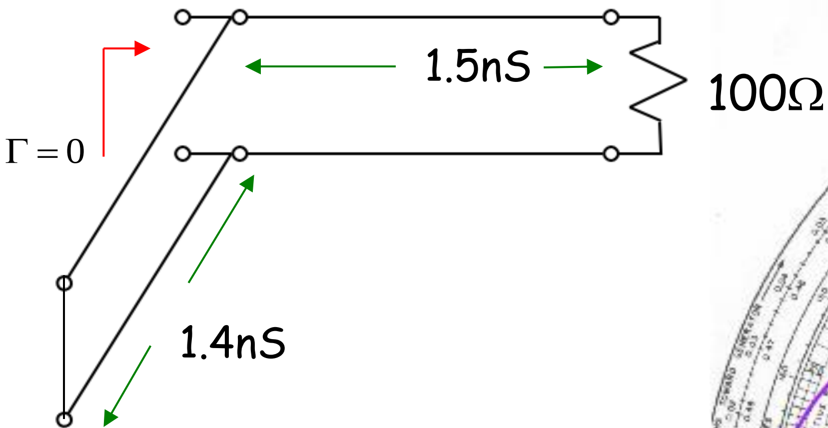
Single Stub Tuner

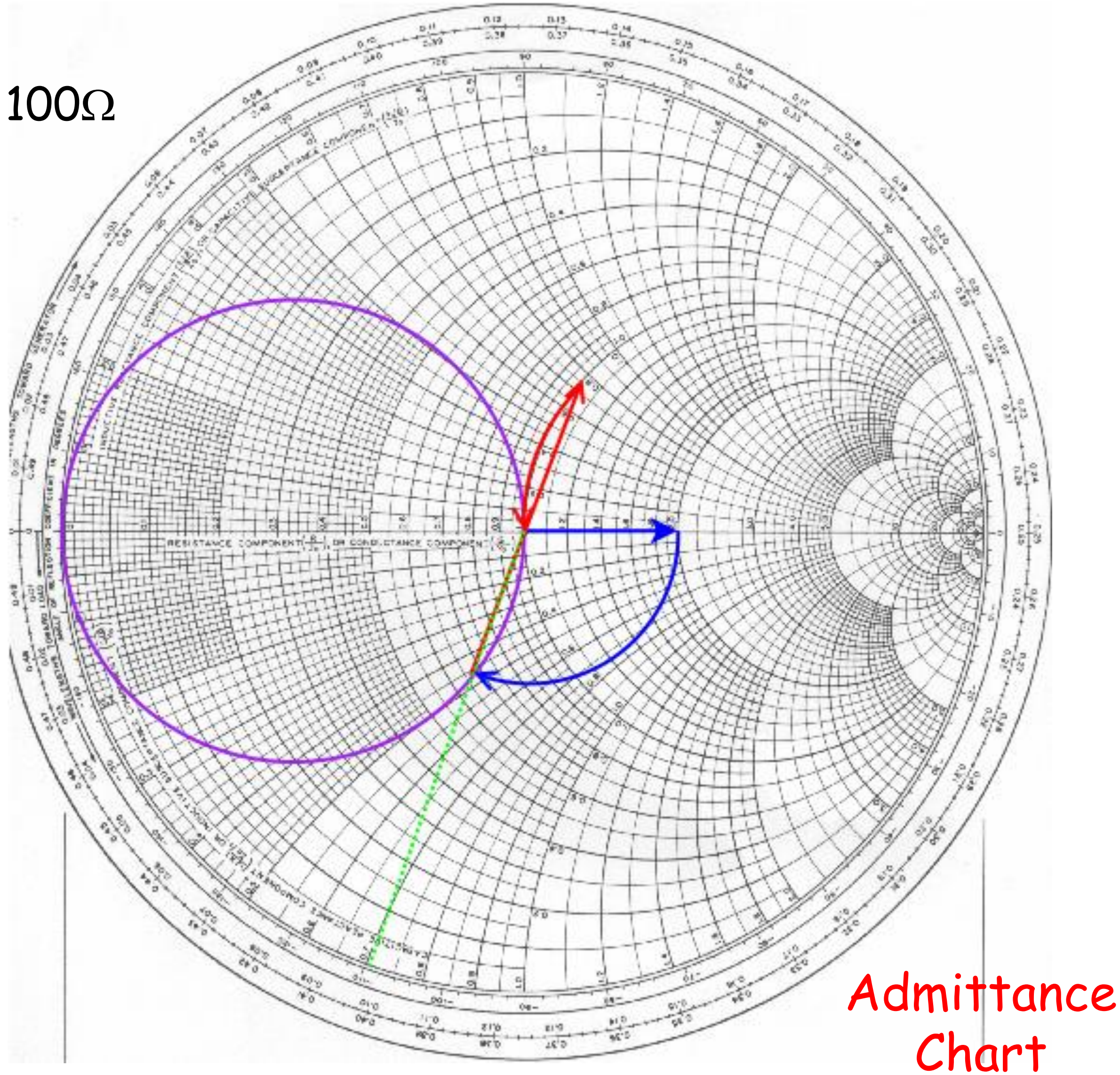




Admittance Chart

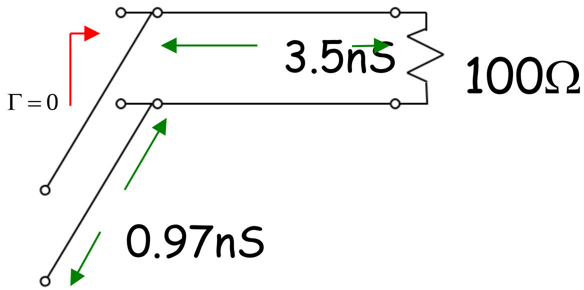
Single Stub Tuner

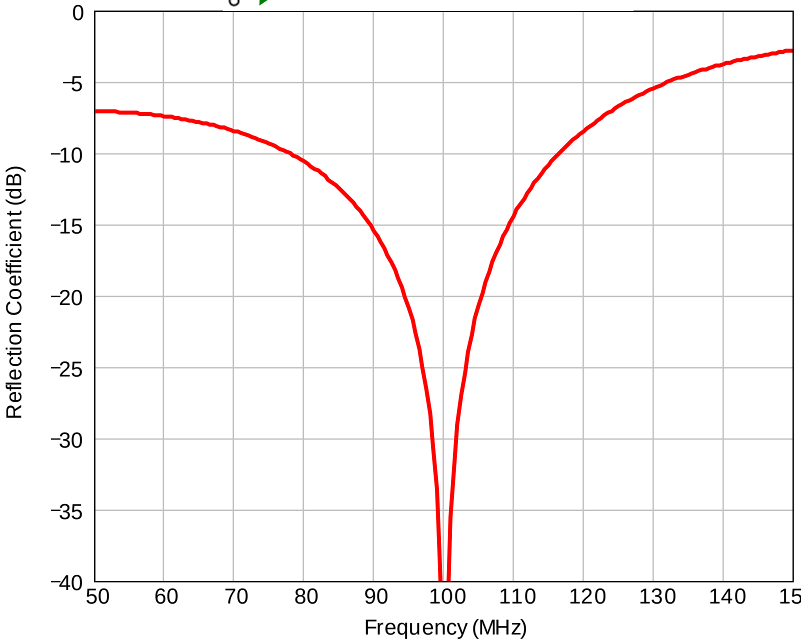


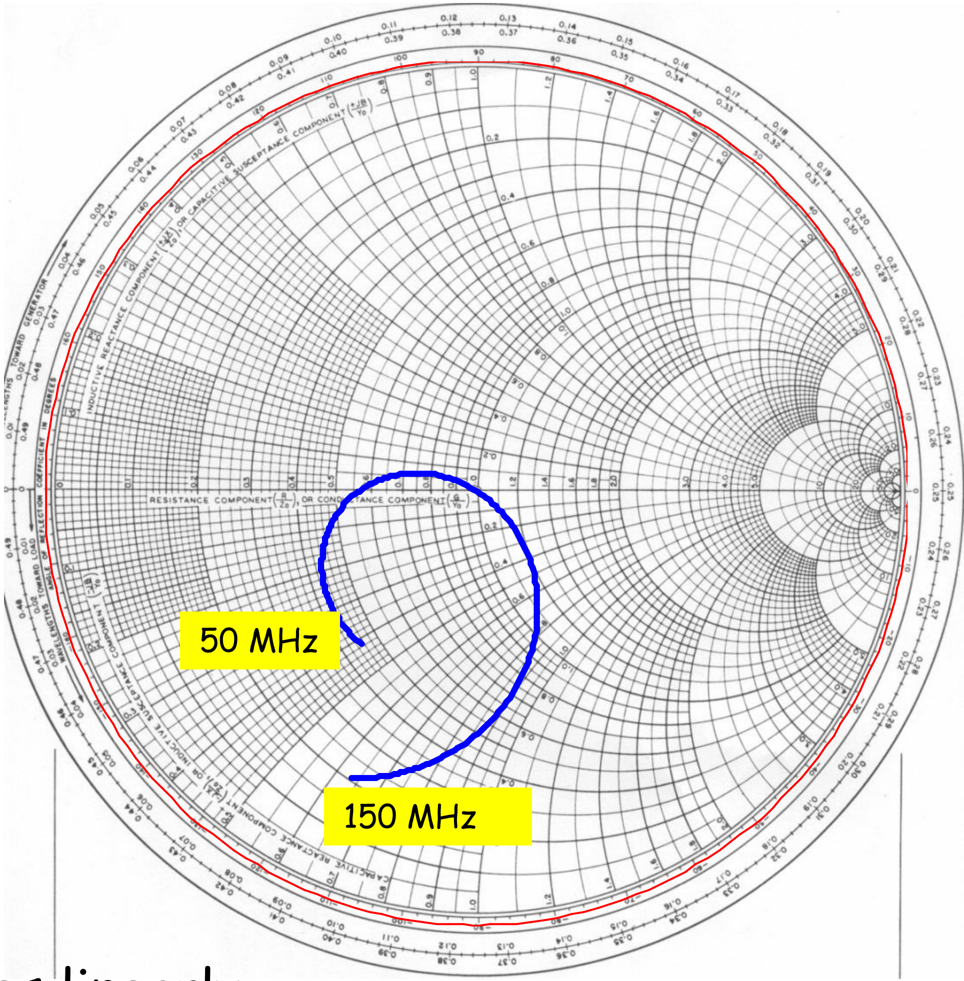


This solution would have worked as well.

Single Stub Tuner Matching Bandwidth





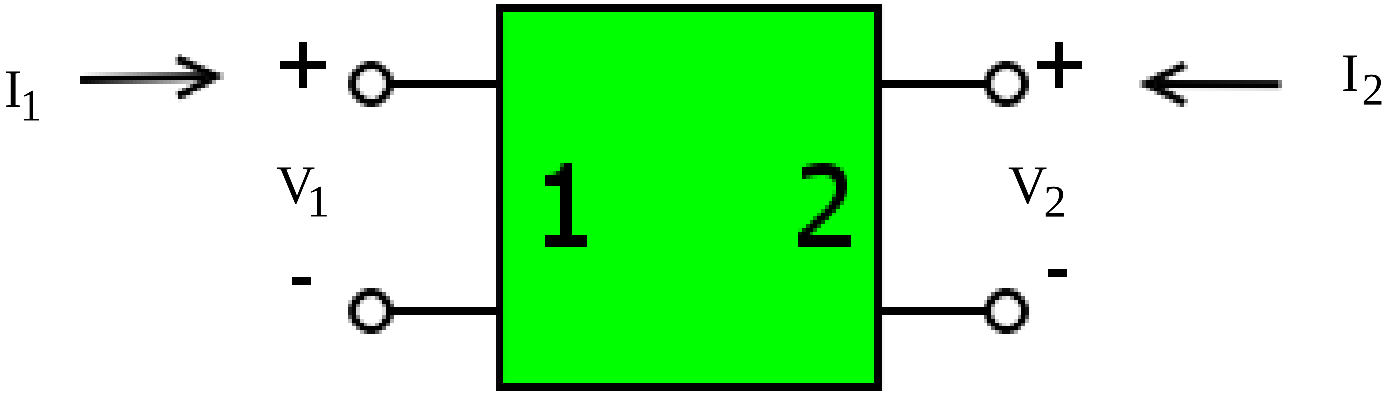


Because the cable phase changes linearly with frequency, the match works over a narrow frequency range

Impedance Chart

Two Port Z Parameters

We have only discussed reflection so far. What about transmission? Consider a device that has two ports:



The device can be characterized by a 2x2 matrix:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$[V] = [Z][I]$$

Scattering (S) Parameters

Since the voltage and current at each port (i) can be broken down into forward and reverse waves:

$$V_i = V_i^+ + V_i^-$$

$$Z_o I_i = V_i^+ - V_i^-$$

We can characterize the circuit with forward and reverse waves:

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

$$\left[V^-\right] = [S]\left[V^+\right]$$

Z and S Parameters

Similar to the reflection coefficient, there is a one-to-one correspondence between the impedance matrix and the scattering matrix:

$$[S] = \left([Z] + Z_o[1]\right)^{-1}\left([Z] - Z_o[1]\right)$$

$$[Z] = Z_o\left([1] + [S]\right)\left([1] - [S]\right)^{-1}$$

Normalized Scattering (S) Parameters

The S matrix defined previously is called the un-normalized scattering matrix. For convenience, define normalized waves:

$$a_i = \frac{V_i^+}{\sqrt{2Z_{o_i}}}$$

$$b_i = \frac{V_i^-}{\sqrt{2Z_{o_i}}}$$

Where Z_{oi} is the characteristic impedance of the transmission line connecting port (i)

$|a_i|^2$ is the forward power into port (i)

$|b_i|^2$ is the reverse power from port (i)

Normalized Scattering (S) Parameters

The normalized scattering matrix is:

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

$$[b] = [s][a]$$

Where:

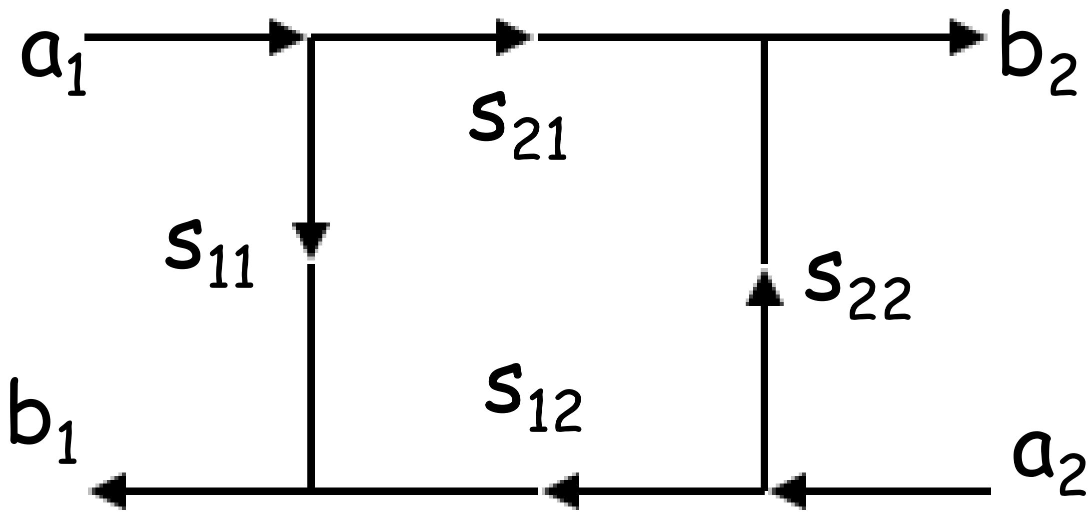
$$s_{i,j} = \sqrt{\frac{Z_{o_j}}{Z_{o_i}}} S_{i,j}$$

If the characteristic impedance on both ports is the same then the normalized and un-normalized S parameters are the same.

Normalized S parameters are the most commonly used.

Normalized S Parameters

The s parameters can be drawn pictorially

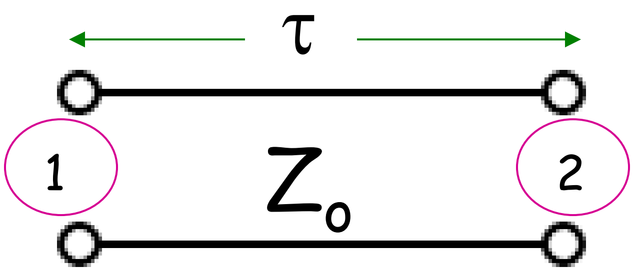


s_{11} and s_{22} can be thought of as reflection coefficients

s_{21} and s_{12} can be thought of as transmission coefficients

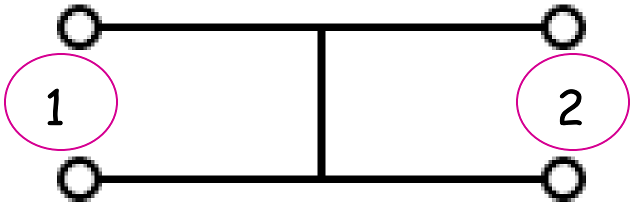
s parameters are complex numbers where the angle corresponds to a phase shift between the forward and reverse waves

Examples of S parameters



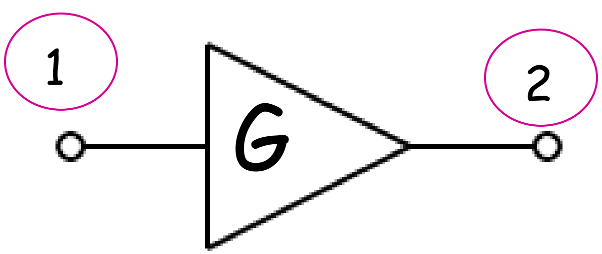
Transmission Line

$$[s] = \begin{bmatrix} 0 & e^{-j\omega\tau} \\ e^{-j\omega\tau} & 0 \end{bmatrix}$$



Short

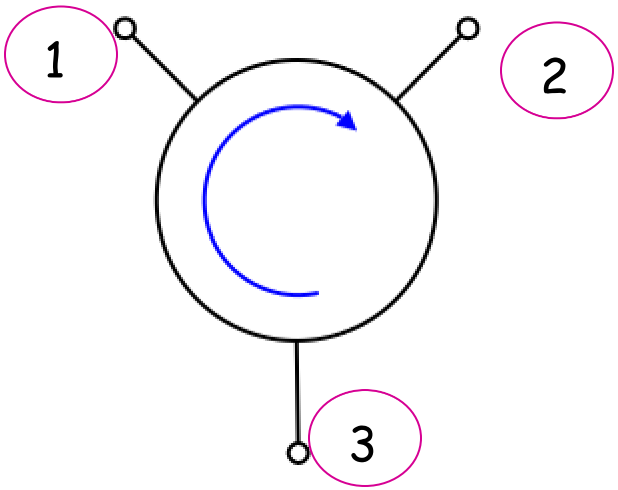
$$[s] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



Amplifier

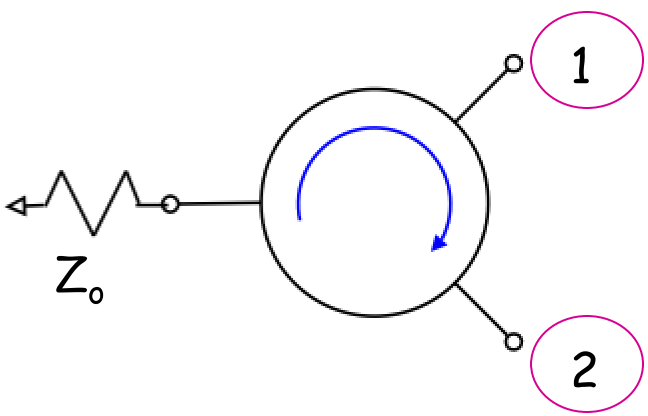
$$[s] = \begin{bmatrix} 0 & 0 \\ G & 0 \end{bmatrix}$$

Examples of S parameters



Circulator

$$[s]=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$



Isolator

$$[s]=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$

Lorentz Reciprocity

If the device is made out of linear isotropic materials (resistors, capacitors, inductors, metal, etc..) then:

$$[S]^{T} = [S]$$

or

$$S_{j,i} = S_{i,j} \qquad \text{for} \quad i \neq j$$

This is equivalent to saying that the transmitting pattern of an antenna is the same as the receiving pattern

reciprocal devices:	transmission line
	short
	directional coupler
non-reciprocal devices:	amplifier
	isolator
	circulator

Lossless Devices

The s matrix of a lossless device is unitary:

$$\left[s^{*}\right]^{T}\left[s\right]=\left[1\right]$$

$$1=\sum_{i}\left|s_{i,j}\right|^{2} \qquad \text{for all j}$$

$$1=\sum_{j}\left|s_{i,j}\right|^{2} \qquad \text{for all i}$$

Lossless devices:
- transmission line
- short
- circulator

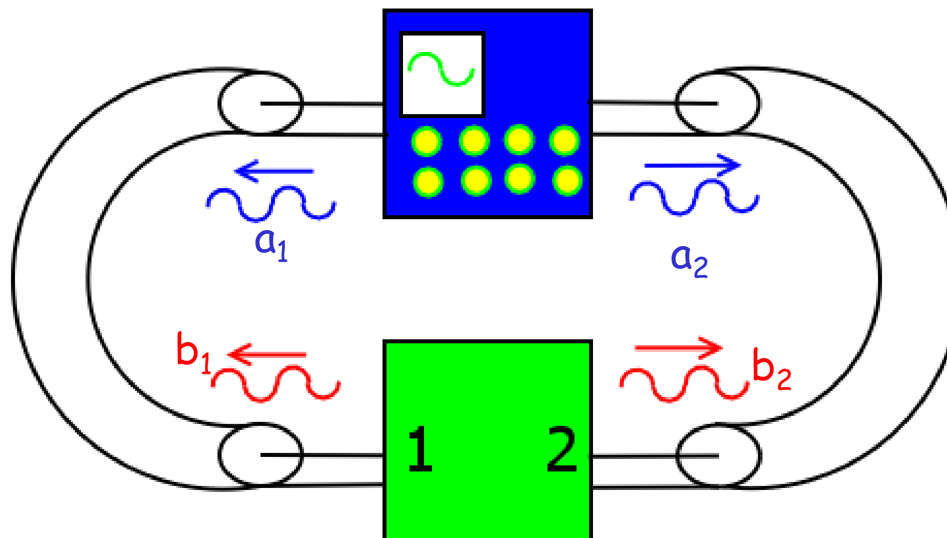
Non-lossless devices:
- amplifier
- isolator

Network Analyzers

- Network analyzers measure S parameters as a function of frequency
- At a **<u>single</u>** frequency, network analyzers send out forward waves a_1 and a_2 and measure the phase and amplitude of the reflected waves b_1 and b_2 with respect to the forward waves.

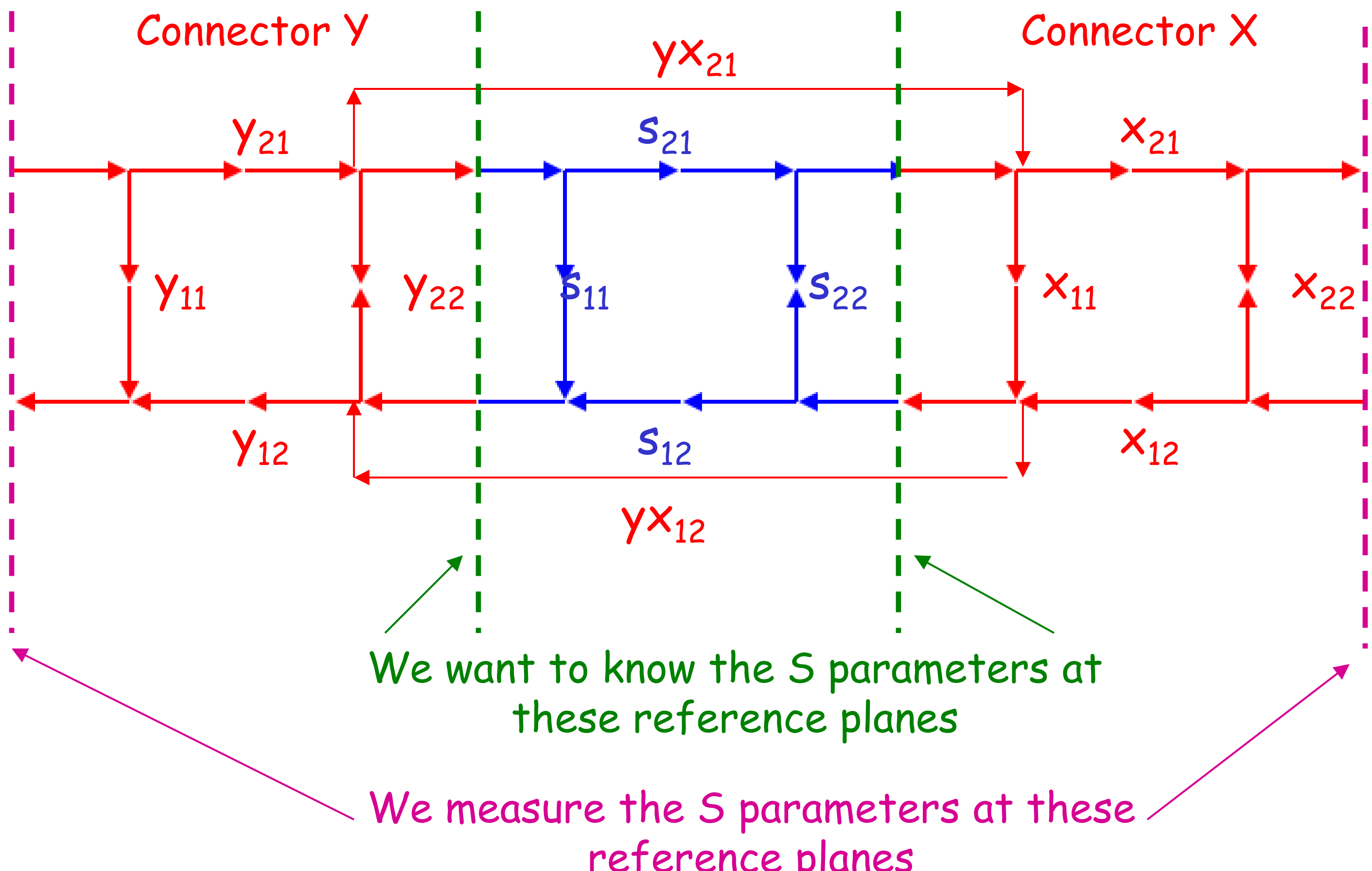
$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \qquad s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$



Network Analyzer Calibration

To measure the pure S parameters of a device, we need to eliminate the effects of cables, connectors, etc. attaching the device to the network analyzer

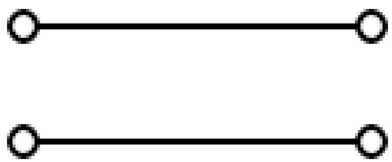


Network Analyzer Calibration

- There are 10 unknowns in the connectors
- We need 10 independent measurements to eliminate these unknowns
  - Develop calibration standards
  - Place the standards in place of the Device Under Test (DUT) and measure the S- parameters of the standards and the connectors
  - Because the S parameters of the calibration standards are known (theoretically), the S parameters of the connectors can be determined and can be mathematically eliminated once the DUT is placed back in the measuring fixtures.

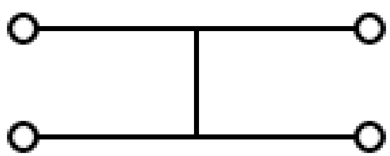
Network Analyzer Calibration

- Since we measure four S parameters for each calibration standard, we need at least three independent standards.
- One possible set is:



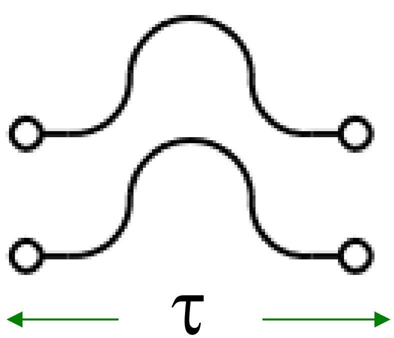
Thru

$$[s] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Short

$$[s] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



Delay*

*$\omega\tau$~90degrees

$$[s] = \begin{bmatrix} 0 & e^{-j\omega\tau} \\ e^{-j\omega\tau} & 0 \end{bmatrix}$$

Phase Delay

A pure sine wave can be written as:

$$V = V_o e^{j(\omega t - \beta z)}$$

The phase shift due to a length of cable is:

$$\theta = \beta d$$

$$= \frac{\omega}{v_{ph}} d$$

$$= \omega \tau_{ph}$$

The <u>phase</u> delay of a device is defined as:

$$\tau_{ph} = -\frac{\arg(S_{21})}{\omega}$$

Phase Delay

- For a non-dispersive cable, the phase delay is the same for all frequencies.
- In general, the phase delay will be a function of frequency.
- It is possible for the phase velocity to take on any value - even greater than the velocity of light
  - Waveguides
  - Waves hitting the shore at an angle

Group Delay

- A pure sine wave has no information content
  - There is nothing changing in a pure sine wave
  - Information is equivalent to something changing
- To send information there must be some modulation of the sine wave at the source

$$V = V_o\left(1 + m\cos(\Delta\omega t)\right)\cos(\omega t)$$

The modulation can be de-composed into different frequency components

$$V = V_o\cos(\omega t) + V_o\frac{m}{2}\left[\cos((\omega+\Delta\omega)t) + \cos((\omega-\Delta\omega)t)\right]$$

Group Delay

The waves emanating from the source will look like

$$V = V_o \cos(\omega t - \beta z)$$
$$+ V_o \frac{m}{2} \cos((\omega + \Delta\omega)t - (\beta + \Delta\beta)z)$$
$$+ V_o \frac{m}{2} \cos((\omega - \Delta\omega)t - (\beta - \Delta\beta)z)$$

Which can be re-written as:

$$V = V_o(1 + m\cos(\Delta\omega t - \Delta\beta z))\cos(\omega t - \beta z)$$

Group Delay

The information travels at a velocity

$$v_{gr} = {}^{1}\!\Big/\!{\frac{\Delta\beta}{\Delta\omega}} \Rightarrow {}^{1}\!\Big/\!{\frac{\partial\beta}{\partial\omega}}$$

The group delay is defined as:

$$\tau_{gr} = \frac{d}{v_{gr}}$$

$$= \frac{\partial\beta}{\partial\omega} d$$

$$= -\frac{\partial(\arg(S_{21}))}{\partial\omega}$$

Phase Delay and Group Delay

Phase Delay:

$$\tau_{ph} = -\frac{\arg(S_{21})}{\omega}$$

Group Delay:

$$\tau_{gr} = -\frac{\partial(\arg(S_{21}))}{\partial\omega}$$

Transmission Line Topics

- Phasors
- Traveling Waves
- Characteristic Impedance
- Reflection Coefficient
- Standing Waves
- Impedance and Reflection
- Incident and Reflected Power
- Smith Charts
- Load Matching
- Single Stub Tuners
- dB and dBm
- Z and S parameters
- Lorentz Reciprocity
- Network Analysis
- Phase and Group Delay